

# **American Community Survey Multiyear Accuracy of the Data (5-year 2012-2016)**

## **INTRODUCTION**

This multiyear ACS Accuracy of the Data document pertains to the 2012-2016 ACS 5 year data products. Due to budget constraints, the 3-year ACS data products have been discontinued. The data contained in these data products are based on the American Community Survey (ACS) sample interviewed from January 1, 2012 through December 31, 2016.

In general, ACS estimates are period estimates that describe the average characteristics of population and housing over a period of data collection. The 2012-2016 ACS 5 year estimates from January 1, 2012 through December 31, 2016. Multiyear estimates give the average value is over the full data period. They cannot be used to say what is going on in any particular year in the period.

The ACS sample is selected from all counties and county-equivalents in the United States. In 2006, the ACS began collection of data from sampled persons in group quarters (GQ) – for example, military barracks, college dormitories, nursing homes, and correctional facilities. Persons in group quarters are included with persons in housing units (HUs) in all 2012-2016 ACS 5 year estimates based on the total population.

The ACS, like any statistical activity, is subject to error. The purpose of this documentation is to provide data users with a basic understanding of the ACS sample design, estimation methodology, and accuracy of the 2012-2016 ACS 5 year estimates. The ACS is sponsored by the U.S. Census Bureau, and is part of the Decennial Census Program.

For additional information on the design and methodology of the ACS, including data collection and processing, visit: <https://www.census.gov/programs-surveys/acs/methodology/design-and-methodology.html>.

For the Multiyear Accuracy of the Data from the Puerto Rico Community Survey, visit: <https://www.census.gov/programs-surveys/acs/technical-documentation/code-lists.html>.

## Table of Contents

<b>INTRODUCTION.....</b>	<b>1</b>
<b>DATA COLLECTION .....</b>	<b>3</b>
<b>SAMPLE DESIGN.....</b>	<b>4</b>
<b>WEIGHTING METHODOLOGY.....</b>	<b>6</b>
<b>ESTIMATION METHODOLOGY FOR MULTIYEAR ESTIMATES.....</b>	<b>7</b>
<b>CONFIDENTIALITY OF THE DATA .....</b>	<b>8</b>
Title 13, United States Code .....	8
Disclosure Avoidance .....	8
Data Swapping .....	8
Synthetic Data .....	9
<b>ERRORS IN THE DATA.....</b>	<b>9</b>
Sampling Error .....	9
Nonsampling Error .....	9
<b>MEASURES OF SAMPLING ERROR.....</b>	<b>10</b>
Confidence Intervals and Margins of Error.....	10
Limitations .....	11
<b>CALCULATION OF STANDARD ERRORS .....</b>	<b>12</b>
Sums and Differences of Direct Standard Errors .....	14
Ratios .....	15
Proportions/Percents .....	15
Percent Change.....	15
Products.....	15
<b>TESTING FOR SIGNIFICANT DIFFERENCES .....</b>	<b>16</b>
<b>EXAMPLES OF STANDARD ERROR CALCULATIONS.....</b>	<b>17</b>
Example 1 – Calculating the Standard Error from the Margin of Error.....	17
Example 2 – Calculating the Standard Error of a Sum or a Difference .....	18
Example 3 – Calculating the Standard Error of a Proportion/Percent.....	18
Example 4 – Calculating the Standard Error of a Ratio .....	19
Example 5 – Calculating the Standard Error of a Product .....	19
<b>CONTROL OF NONSAMPLING ERROR.....</b>	<b>20</b>
Coverage Error .....	21
Nonresponse Error .....	22
Measurement and Processing Error.....	23
<b>ISSUES WITH APPROXIMATING THE STANDARD ERROR OF LINEAR COMBINATIONS OF MULTIPLE ESTIMATES .....</b>	<b>25</b>

## DATA COLLECTION

The ACS employs four modes of data collection:

1. Internet
2. Mailout/Mailback
3. Computer Assisted Telephone Interview (CATI)
4. Computer Assisted Personal Interview (CAPI)

The general timing of data collection is as follows. Note that mail responses are accepted during all three months of data collection:

Month 1: Prior to 2013, addresses that were determined to be mailable were sent questionnaires via the U.S. Postal Service. This included a second questionnaire, in case a sample address had not responded within approximately three weeks after having received the initial questionnaire.

From 2013 onward, addresses in sample that are determined to be mailable are sent an initial mailing package – this package contains information for completing the ACS questionnaire on the internet (on-line). If, after two weeks, a sample address has not responded on-line, then it is sent a second mailing package. This package contains a paper questionnaire. Once the second package is received, sampled addresses then have the option of which mode to use for filling out the questionnaire.

Month 2: All mail non-responding addresses with an available phone number are sent to CATI.

Month 3: A sample of mail non-responses without a phone number, CATI non-responses, and unmailable addresses are selected and sent to CAPI.

## SAMPLE DESIGN

Sampling rates are assigned independently at the census block level. A measure of size is calculated for each of the following governmental units:

- Counties
- Places
- School Districts (elementary, secondary, and unified)
- American Indian Areas
- Tribal Subdivisions
- Alaska Native Village Statistical Areas
- Hawaiian Homelands
- Minor Civil Divisions – in Connecticut, Maine, Massachusetts, Michigan, Minnesota, New Hampshire, New Jersey, New York, Pennsylvania, Rhode Island, Vermont, and Wisconsin (these are the states where MCDs are active, functioning governmental units)
- Census Designated Places – in Hawaii only

The measure of size for all areas except American Indian Areas is an estimate of the number of occupied HUs in the area. This is calculated by multiplying the number of ACS addresses by an estimated occupancy rate at the block level. A measure of size for each Census Tract is also calculated in the same manner.

For American Indian Areas, the measure of size is the estimated number of occupied HUs multiplied by the proportion of people reporting American Indian alone or in combination in the Census. Prior to 2011, this was calculated using Census 2000 occupancy and response data. Beginning in 2011, the ACS uses occupancy and response data from the 2010 Census.

Each block is then assigned the smallest measure of size from the set of all entities of which it is a part. Sampling rates for 2012, 2013, 2014, 2015, and 2016 are shown in Table 1. Beginning in 2011 the ACS implemented a sample reallocation as well as a sample increase. The sample reallocation increased the number of second-stage sampling strata from seven to 16. The sample increase changed the target annual sample size from 2.9 million to 3.54 million.

**Table 1. 2012 through 2016 Sampling Rates for the United States**

Stratum Thresholds	2012 Sampling Rates	2013 Sampling Rates	2014 Sampling Rates	2015 Sampling Rates	2016 Sampling Rates
0 < MOS <sup>1</sup> ≤ 200	15.00%	15.00%	15.00%	15.00%	15.00%
200 < MOS ≤ 400	10.00%	10.00%	10.00%	10.00%	10.00%
400 < MOS ≤ 800	7.00%	7.00%	7.00%	7.00%	7.00%
800 < MOS < 1,200	4.40%	4.40%	4.37%	4.35%	4.34%
0 < TRACTMOS <sup>2</sup> ≤ 400	5.55%	5.55%	5.47%	5.44%	5.42%
0 < TRACTMOS ≤ 400 H.R. <sup>3</sup>	5.06%	5.06%	5.03%	5.00%	4.99%
400 < TRACTMOS ≤ 1,000	4.40%	4.40%	4.37%	4.35%	4.34%
400 < TRACTMOS ≤ 1,000 H.R.	4.04%	4.04%	4.02%	4.00%	3.99%
1,000 < TRACTMOS ≤ 2,000	2.67%	2.67%	2.66%	2.64%	2.63%
1,000 < TRACTMOS ≤ 2,000 H.R.	2.46%	2.46%	2.44%	2.43%	2.42%
2,000 < TRACTMOS ≤ 4,000	1.57%	1.57%	1.56%	1.55%	1.55%
2,000 < TRACTMOS ≤ 4,000 H.R.	1.44%	1.44%	1.44%	1.43%	1.43%
4,000 < TRACTMOS ≤ 6,000	0.94%	0.94%	0.94%	0.93%	0.93%
4,000 < TRACTMOS ≤ 6,000 H.R.	0.87%	0.87%	0.86%	0.86%	0.86%
6,000 < TRACTMOS	0.55%	0.55%	0.55%	0.54%	0.54%
6,000 < TRACTMOS H.R.	0.51%	0.51%	0.50%	0.50%	0.50%

<sup>1</sup>MOS = Measure of size of the smallest governmental entity

<sup>2</sup>TRACTMOS = Census Tract measure of size.

<sup>3</sup>H.R. = areas where predicted levels of completed mail and CATI interviews are > 60%.

Prior to 2011, all addresses determined to be unmailable were subsampled for the CAPI phase of data collection at a rate of 2-in-3. All other sample addresses that did not respond to the mail or CATI modes of data collections were subsampled for CAPI at a rate of 1-in-2, 2-in-5 or 2-in-3. Unmailable addresses, which include Remote Alaska addresses, do not go to the CATI phase of data collection. Beginning in 2011, at the same time as the sample increase, there was a change in how addresses were selected for CAPI. Table 2 shows the new CAPI sampling rates that were implemented in 2011 (midway through the year).

**Table 2. 2012 Through 2016 CAPI Subsampling Rates for the United States**

Address and Tract Characteristics	CAPI Subsampling Rate
Addresses in Remote Alaska	100%
Addresses in Hawaiian Homelands, Alaska Native Village Statistical areas and a subset of American Indian areas	100%
Unmailable addresses that are not in the previous two categories	66.7%
Mailable addresses in tracts with predicted levels of completed mail and CATI interviews prior to CAPI subsampling between 0% and less than 36%	50.0%
Mailable addresses in tracts with predicted levels of completed mail and CATI interviews prior to CAPI subsampling greater than 35% and less than 51%	40.0%
Mailable addresses in other tracts	33.3%

For a more detailed description of the ACS sampling methodology, see the ACS Accuracy of the Data document. This document is available for 2016 as well as prior data years at: <https://www.census.gov/programs-surveys/acs/technical-documentation/code-lists.html>.

## WEIGHTING METHODOLOGY

The multiyear estimates should be interpreted as estimates that describe a time period rather than a specific reference year. For example, a 5-year estimate for the poverty rate of a given area describes the total set of people who lived in that area over those five years much the same way as a 1-year estimate for the same characteristic describes the set of people who lived in that area over one year. The only fundamental difference between the estimates is the number of months of collected data, which are considered in forming the estimate. For this reason, the estimation procedure used for the multiyear estimates is an extension of the 2016 1-year estimation procedure. In this document only the procedures that are unique to the multiyear estimates are discussed.

To weight the 5-year estimates, 60 months of collected data are pooled together. The pooled data are then reweighted using the procedures developed for the 2016 1-year estimates with a few adjustments. These adjustments concern geography, month-specific weighting steps, and population and housing unit controls. In addition to these adjustments, there is one multiyear specific model-assisted weighting step.

Some of the weighting steps use the month of tabulation in forming the weighting cells within which the weighting adjustments are made. One such example is the non-interview adjustment. In these weighting steps, the month of tabulation is used independently of year. Thus, for the 5-year, sample cases from May 2012, May 2013, May 2014, May 2015, and May 2016 are combined.

Since the multiyear estimates represent estimates for the period, the controls are not a single year's housing or population estimates from the Population Estimates Program, but rather are an average of these estimates over the period. For the housing unit controls, a simple average of the 1-year housing unit estimates over the period is calculated for each county or subcounty area. The version or vintage of estimates used is always the last year of the period since these are considered the most up-to-date and are created using a consistent methodology. For example, the housing unit control used for a given county in the 2012-2016 weighting is equal to the simple average of the 2012, 2013, 2014, 2015, and 2016 estimates that were produced using the 2016 methodology (the 2016 vintage). Likewise, the population controls by race, ethnicity, age, and sex are obtained by taking a simple average of the 1-year population estimates of the county or weighting area by race, ethnicity, age, and sex. For example, the 2012-2016 control total used for Hispanic males age 20-24 in a given county would be obtained by averaging the 1-year population estimates for that demographic group for 2012, 2013, 2014, 2015 and 2016. The version or vintage of estimates used is always that of the last year of the period since these are considered the most up to date and are created using a consistent methodology.

One multiyear specific step is a model-assisted (generalized regression or GREG) weighting step. The objective of this additional step is to reduce the variances of base demographics at the tract level in the 5-year estimates. While reducing the variances, the estimates themselves are relatively unchanged. This process involves linking administrative record data with ACS data.

The GQ weighting methodology imputes GQ person records into the 2012-2016 ACS 5-year. See the American Community Survey Accuracy of the Data (2016) for details on the GQ imputation.

In addition, a finite population correction (FPC) factor is included in the creation of the replicate weights for the 5-year data at the tract level. It reduces the estimate of the variance and the margin of error by taking the sampling rate into account. A two-tiered approach is used. One FPC is calculated for mail, internet, and CATI respondents and another for CAPI respondents. The CAPI is given a separate FPC to take into account the fact that CAPI respondents are subsampled. The FPC is not included in the 1-year data because the sampling rates are relatively small and thus the FPC does not have an appreciable impact on the variance.

For more information on the replicate weights and replicate factors, see the Design and Methodology Report at: <https://www.census.gov/programs-surveys/acs/methodology/design-and-methodology.html>

## **ESTIMATION METHODOLOGY FOR MULTIYEAR ESTIMATES**

For the 1-year estimation, the tabulation geography for the data is based on the boundaries defined on January 1 of the tabulation year, which is consistent with the tabulation geography used to produce the population estimates. All sample addresses are updated with this geography prior to weighting. For the multiyear estimation, the tabulation geography for the data is referenced to the final year in the multiyear period. For example, the 2012-2016 period uses the 2016 reference geography. Thus, all data collected over the period of 2012-2016 in the blocks that are contained in the 2016 boundaries for a given place are tabulated as though they were a part of that place for the entire period.

Monetary values for the ACS multiyear estimates are inflation-adjusted to the final year of the period. For example, the 2012-2016 ACS 5-year estimates are tabulated using 2016-adjusted dollars. These adjustments use the national Consumer Price Index (CPI) since a regional-based CPI is not available for the entire country.

For a more detailed description of the ACS estimation methodology, see the Accuracy of the Data document. This document is available for 2016 and prior data years at: <https://www.census.gov/programs-surveys/acs/technical-documentation/code-lists.html>

## **CONFIDENTIALITY OF THE DATA**

The Census Bureau has modified or suppressed some data on this site to protect confidentiality. Title 13 United States Code, Section 9, prohibits the Census Bureau from publishing results in which an individual's data can be identified.

The Census Bureau's internal Disclosure Review Board sets the confidentiality rules for all data releases. A checklist approach is used to ensure that all potential risks to the confidentiality of the data are considered and addressed.

### **Title 13, United States Code**

Title 13 of the United States Code authorizes the Census Bureau to conduct censuses and surveys. Section 9 of the same Title requires that any information collected from the public under the authority of Title 13 be maintained as confidential. Section 214 of Title 13 and Sections 3559 and 3571 of Title 18 of the United States Code provide for the imposition of penalties of up to five years in prison and up to \$250,000 in fines for wrongful disclosure of confidential census information.

### **Disclosure Avoidance**

Disclosure avoidance is the process for protecting the confidentiality of data. A disclosure of data occurs when someone can use published statistical information to identify an individual who has provided information under a pledge of confidentiality. For data tabulations, the Census Bureau uses disclosure avoidance procedures to modify or remove the characteristics that put confidential information at risk for disclosure. Although it may appear that a table shows information about a specific individual, the Census Bureau has taken steps to disguise or suppress the original data while making sure the results are still useful. The techniques used by the Census Bureau to protect confidentiality in tabulations vary, depending on the type of data. All disclosure avoidance procedures are done prior to the whole person imputation into not-in-sample GQ facilities.

### **Data Swapping**

Data swapping is a method of disclosure avoidance designed to protect confidentiality in tables of frequency data (the number or percent of the population with certain characteristics). Data swapping is done by editing the source data or exchanging records for a sample of cases when creating a table. A sample of households is selected and matched on a set of selected key variables with households in neighboring geographic areas that have similar characteristics (such as the same number of adults and same number of children). Because the swap often occurs within a neighboring area, there is no effect on the marginal totals for the area or for totals that include data from multiple areas. Because of data swapping, users should not assume that tables with cells having a value of one or two reveal information about specific individuals. Data swapping procedures were first used in the 1990 Census, and were used again in Census 2000 and the 2010 Census.

## **Synthetic Data**

The goals of using synthetic data are the same as the goals of data swapping, namely to protect the confidentiality in tables of frequency data. Persons are identified as being at risk for disclosure based on certain characteristics. The synthetic data technique then models the values for another collection of characteristics to protect the confidentiality of that individual.

Note: The data use the same disclosure avoidance methodology as the original 1-year data. The confidentiality edit was previously applied to the raw data files when they were created to produce the 1-year estimates and these same data files with the original confidentiality edit were used to produce the 5-year estimates.

## **ERRORS IN THE DATA**

### **Sampling Error**

The data in ACS products are estimates of the actual figures that would be obtained by interviewing the entire population. The estimates are a result of the chosen sample, and are subject to sample-to-sample variation. Sampling error in data arises due to the use of probability sampling, which is necessary to ensure the integrity and representativeness of sample survey results. The implementation of statistical sampling procedures provides the basis for the statistical analysis of sample data. Measures used to estimate the sampling error are provided in the next section.

### **Nonsampling Error**

Other types of errors might be introduced during any of the various complex operations used to collect and process survey data. For example, data entry from questionnaires and editing may introduce error into the estimates. Another potential source of error is the use of controls in the weighting. These controls are based on Population Estimates and are designed to reduce variance and mitigate the effects of systematic undercoverage of groups who are difficult to enumerate. However, if the extrapolation methods used in generating the Population Estimates do not properly reflect the population, error can be introduced into the data. This potential risk is offset by the many benefits the controls provide to the ACS estimates, which include the reduction of issues with survey coverage and the reduction of standard errors of ACS estimates. These and other sources of error contribute to the nonsampling error component of the total error of survey estimates.

Nonsampling errors may affect the data in two ways. Errors that are introduced randomly increase the variability of the data. Systematic errors, or errors that consistently skew the data in one direction, introduce bias into the results of a sample survey. The Census Bureau protects against the effect of systematic errors on survey estimates by conducting extensive research and evaluation programs on sampling techniques, questionnaire design, and data collection and processing procedures.

An important goal of the ACS is to minimize the amount of nonsampling error introduced through nonresponse for sample housing units. One way of accomplishing this is by following up on mail nonrespondents during the CATI and CAPI phases. For more information, please see the section entitled “Control of Nonsampling Error”.

## MEASURES OF SAMPLING ERROR

Sampling error is the difference between an estimate based on a sample and the corresponding value that would be obtained if the entire population were surveyed (as for a census). Note that sample-based estimates will vary depending on the particular sample selected from the population. Measures of the magnitude of sampling error reflect the variation in the estimates over all possible samples that could have been selected from the population using the same sampling methodology.

Estimates of the magnitude of sampling errors – in the form of margins of error – are provided with all published ACS data. The Census Bureau recommends that data users incorporate margins of error into their analyses, as sampling error in survey estimates could impact the conclusions drawn from the results.

### Confidence Intervals and Margins of Error

#### Confidence Intervals

A sample estimate and its estimated standard error may be used to construct confidence intervals about the estimate. These intervals are ranges that will contain the average value of the estimated characteristic that results over all possible samples, with a known probability.

For example, if all possible samples that could result under the ACS sample design were independently selected and surveyed under the same conditions, and if the estimate and its estimated standard error were calculated for each of these samples, then:

1. Approximately 68 percent of the intervals from one estimated standard error below the estimate to one estimated standard error above the estimate would contain the average result from all possible samples.
2. Approximately 90 percent of the intervals from 1.645 times the estimated standard error below the estimate to 1.645 times the estimated standard error above the estimate would contain the average result from all possible samples.
3. Approximately 95 percent of the intervals from two estimated standard errors below the estimate to two estimated standard errors above the estimate would contain the average result from all possible samples.

The intervals are referred to as 68 percent, 90 percent, and 95 percent confidence intervals, respectively.

### **Margins of Error**

In lieu of providing upper and lower confidence bounds in published ACS tables, the margin of error is listed. All ACS published margins of error are based on a 90 percent confidence level. The margin of error is the difference between an estimate and its upper or lower confidence bound. Both the confidence bounds and the standard error can easily be computed from the margin of error:

$$\text{Standard Error} = \text{Margin of Error} / 1.645$$

$$\text{Lower Confidence Bound} = \text{Estimate} - \text{Margin of Error}$$

$$\text{Upper Confidence Bound} = \text{Estimate} + \text{Margin of Error}$$

Note that for 2005 and earlier estimates, ACS margins of error and confidence bounds were calculated using a 90 percent confidence level multiplier of 1.65. Starting with the 2006 data release, and for every year after 2006, the more accurate multiplier of 1.645 is used. Margins of error and confidence bounds from previously published products will not be updated with the new multiplier. When calculating standard errors from margins of error or confidence bounds using published data for 2005 and earlier, use the 1.65 multiplier.

When constructing confidence bounds from the margin of error, users should be aware of any “natural” limits on the bounds. For example, if a characteristic estimate for the population is near zero, the calculated value of the lower confidence bound may be negative. However, as a negative number of people does not make sense, the lower confidence bound should be reported as zero. For other estimates such as income, negative values can make sense; in these cases, the lower bound should not be adjusted. The context and meaning of the estimate must therefore be kept in mind when creating these bounds. Another example of a natural limit is 100 percent as the upper bound of a percent estimate.

If the margin of error is displayed as ‘\*\*\*\*\*’ (five asterisks), the estimate has been controlled to be equal to a fixed value and so it has no sampling error. A standard error of zero should be used for these controlled estimates when completing calculations, such as those in the following section.

### **Limitations**

Users should be careful when computing and interpreting confidence intervals.

## Nonsampling Error

The estimated standard errors (and thus margins of error) included in these data products do not account for variability due to nonsampling error that may be present in the data. In particular, the standard errors do not reflect the effect of correlated errors introduced by interviewers, coders, or other field or processing personnel or the effect of imputed values due to missing responses. The standard errors calculated are only lower bounds of the total error. As a result, confidence intervals formed using these estimated standard errors may not meet the stated levels of confidence (i.e., 68, 90, or 95 percent). Some care must be exercised in the interpretation of the data based on the estimated standard errors.

## Very Small (Zero) or Very Large Estimates

By definition, the value of almost all ACS characteristics is greater than or equal to zero. The method provided above for calculating confidence intervals relies on large sample theory, and may result in negative values for zero or small estimates for which negative values are not admissible. In this case, the lower limit of the confidence interval should be set to zero by default. A similar caution holds for estimates of totals close to a control total or estimated proportion near one, where the upper limit of the confidence interval is set to its largest admissible value. In these situations, the level of confidence of the adjusted range of values is less than the prescribed confidence level.

## CALCULATION OF STANDARD ERRORS

Direct estimates of margin of error were calculated for all estimates reported. The margin of error is derived from the variance. In most cases, the variance is calculated using a replicate-based methodology known as successive difference replication that takes into account the sample design and estimation procedures.

The formula provided below calculates the variance using the ACS estimate ( $X_0$ ) and the 80 replicate estimates ( $X_r$ ).

$$\text{Variance} = \frac{4}{80} \sum_{r=1}^{80} (x_r - x_0)^2$$

$X_0$  is the estimate calculated using the production weight and  $X_r$  is the estimate calculated using the  $r^{\text{th}}$  replicate weight. The standard error is the square root of the variance. The 90<sup>th</sup> percent margin of error is 1.645 times the standard error.

Additional information on the formation of the replicate weights, can be found in Chapter 12 of the Design and Methodology documentation at:

<https://www.census.gov/programs-surveys/acs/methodology/design-and-methodology.html>.

Beginning with the 2011 ACS 1-year estimates, a new imputation-based methodology was incorporated into processing (see the description in the Group Quarters Person Weighting Section). An adjustment was made to the production replicate weight variance methodology to account for the non-negligible amount of additional variation being introduced by the new technique.<sup>1</sup>

Excluding the base weights, replicate weights were allowed to be negative in order to avoid underestimating the standard error. Exceptions include:

1. *The estimate of the number or proportion of people, households, families, or housing units in a geographic area with a specific characteristic is zero.* A special procedure is used to estimate the standard error.
2. *There are either no sample observations available to compute an estimate or standard error of a median, an aggregate, a proportion, or some other ratio, or there are too few sample observations to compute a stable estimate of the standard error.* The estimate is represented in the tables by “-” and the margin of error by “\*\*” (two asterisks).
3. *The estimate of a median falls in the lower open-ended interval or upper open-ended interval of a distribution.* If the median occurs in the lowest interval, then a “-” follows the estimate, and if the median occurs in the upper interval, then a “+” follows the estimate. In both cases, the margin of error is represented in the tables by “\*\*\*” (three asterisks).

## Calculating Measures of Error Using Variance Replicate Tables

Advanced users may be interested in the Variance Replicate Tables, first released for the 2010-2014 ACS 5-year data in July 2016. These augmented ACS Detailed Tables include sets of 80 replicate estimates, which allow users to calculate measures of error for derived estimates using the same methods that are used to produce the published MOEs on AFF. These methods incorporate the covariance between estimates that the approximation formulas in this document leave out.

The Variance Replicate Tables are available for a subset of the 5-year Detailed Tables for eleven summary levels, including the nation, states, counties, tracts, and block groups. These will be released on an annual basis, shortly after the release of the regular 5-year data products.

---

<sup>1</sup> For more information regarding this issue, see Asiala, M. and Castro, E. 2012. Developing Replicate Weight-Based Methods to Account for Imputation Variance in a Mass Imputation Application. In JSM proceedings, Section on Survey Research Methods, Alexandria, VA: American Statistical Association.

The Variance Replicate Tables can be found at: <https://census.gov/programs-surveys/acs/data/variance-tables.html>

The Variance Replicate Documentation (including table list and summary level list) can be found at: <https://census.gov/programs-surveys/acs/technical-documentation/variance-tables.html>

## Sums and Differences of Direct Standard Errors

Estimates of standard errors displayed in tables are for individual estimates. Additional calculations are required to estimate the standard errors for sums of or the differences between two or more sample estimates.

The standard error of the sum of two sample estimates is the square root of the sum of the two individual standard errors squared plus a covariance term. That is, for standard errors  $SE(\hat{X}_1)$  and  $SE(\hat{X}_2)$  of estimates  $\hat{X}_1$  and  $\hat{X}_2$ :

$$SE(\hat{X}_1 \pm \hat{X}_2) = \sqrt{[SE(\hat{X}_1)]^2 + [SE(\hat{X}_2)]^2 \pm 2\text{cov}(\hat{X}_1, \hat{X}_2)} \quad (1)$$

The covariance measures the interactions between two estimates. Currently the covariance terms are not available. Data users should therefore use the following approximation:

$$SE(\hat{X}_1 \pm \hat{X}_2) = \sqrt{[SE(\hat{X}_1)]^2 + [SE(\hat{X}_2)]^2} \quad (2)$$

However, it should be noted that this approximation will underestimate or overestimate the standard error if the two estimates interact in either a positive or a negative way.

The approximation formula (2) can be expanded to more than two estimates by adding in the individual standard errors squared inside the radical. As the number of estimates involved in the sum or difference increases, the results of the approximation become increasingly different from the standard error derived directly from the ACS microdata. Care should be taken to work with the fewest number of estimates as possible. If there are estimates involved in the sum that are controlled, then the approximate standard error can be increasingly different. Later in this document, examples are provided to demonstrate issues associated with approximating the standard errors when summing large numbers of estimates together.

## Ratios

The statistic of interest may be the ratio of two estimates, where the numerator *is not* a subset of the denominator. The standard error of this ratio between two sample estimates is approximated as:

$$SE\left(\frac{\hat{X}}{\hat{Y}}\right) = \frac{1}{\hat{Y}} \sqrt{[SE(\hat{X})]^2 + \frac{\hat{X}^2}{\hat{Y}^2} [SE(\hat{Y})]^2} \quad (3)$$

## Proportions/Percents

For a proportion (or percent), a ratio where the numerator *is* a subset of the denominator, a slightly different estimator is used. If  $\hat{P} = \hat{X}/\hat{Y}$  then the standard error of this proportion is approximated as:

$$SE(\hat{P}) = \frac{1}{\hat{Y}} \sqrt{[SE(\hat{X})]^2 - \frac{\hat{X}^2}{\hat{Y}^2} [SE(\hat{Y})]^2} \quad (4)$$

If  $\hat{Q} = 100\% * \hat{P}$  (P is the proportion and Q is its corresponding percent), then

$SE(\hat{Q}) = 100\% * SE(\hat{P})$ . Note the difference between the formulas to approximate the standard error for proportions (4) and ratios (3) - the plus sign in the previous formula has been replaced with a minus sign. If the value under the radical is negative, use the ratio standard error formula instead.

## Percent Change

The statistic of interest is a percentage change from one time period to another, where the more current estimate is compared to an older estimate, for example, the percent change of a 2005-2009 estimate to a 2012-2016 estimate. If the current estimate =  $\hat{X}$  and the earlier estimate =  $\hat{Y}$ , then the standard error for the percent change is approximated as:

$$SE\left(\frac{\hat{X} - \hat{Y}}{\hat{Y}}\right) = SE\left(\frac{\hat{X}}{\hat{Y}} - 1\right) = SE\left(\frac{\hat{X}}{\hat{Y}}\right) \quad (5)$$

As a caveat, this formula does not take into account the correlation when calculating overlapping time periods.

## Products

For a product of two estimates - for example, deriving a proportion's numerator by multiplying the proportion by its denominator - the standard error can be approximated as:

$$SE(\hat{X} \times \hat{Y}) = \sqrt{\hat{X}^2 \times [SE(\hat{Y})]^2 + \hat{Y}^2 \times [SE(\hat{X})]^2} \quad (6)$$

## TESTING FOR SIGNIFICANT DIFFERENCES

Users may conduct a statistical test to see if the difference between an ACS estimate and any other chosen estimate is statistically significant at a given confidence level. “Statistically significant” means that it is not likely that the difference between estimates is due to random chance alone.

To perform statistical significance testing, first calculate a Z statistic from the two estimates ( $Est_1$  and  $Est_2$ ) and their respective standard errors ( $SE_1$  and  $SE_2$ ):

$$Z = \frac{Est_1 - Est_2}{\sqrt{(SE_1)^2 + (SE_2)^2}} \quad (7)$$

If  $Z > 1.645$  or  $Z < -1.645$ , then the difference can be said to be statistically significant at the 90 percent confidence level.<sup>2</sup>

Any pair of estimates can be compared using this method, including ACS estimates from the current year, ACS estimates from a previous year, 2010 Census counts, estimates from other Census Bureau surveys, and estimates from other sources.

Note: Not all estimates are subject to sampling error. For example: Census 2010 counts and Census 2000 100 percent counts are not. However, Census 2000 long form estimates, estimates from other surveys, and, as discussed throughout this document, ACS estimates are subject to sampling error and should have some measure of this error available to users. Measures include margins of error, standard errors, or confidence intervals. If these measures are available, they should be used to give the most accurate result of the test.

Users are also cautioned to *not* rely on looking at whether confidence intervals for two estimates overlap in order to determine statistical significance. There are circumstances where comparing confidence intervals will not give the correct test result. If two confidence intervals do not overlap, then the estimates will be significantly different (i.e. the significance test will always agree). However, if two confidence intervals do overlap, then the estimates may or

---

<sup>2</sup> The ACS Accuracy of the Data document in 2005 used a Z statistic of +/-1.65. Data users should use +/-1.65 for estimates published in 2005 or earlier.

may not be significantly different. The Z calculation shown above is recommended in all cases.

The following example illustrates why using the overlapping confidence bounds rule of thumb as a substitute for a statistical test is not recommended.

Let:  $X_1 = 6.0$  with  $SE_1 = 0.5$  and  $X_2 = 5.0$  with  $SE_2 = 0.2$ .

The Lower Bound for  $X_1 = 6.0 - 0.5 * 1.645 = 5.2$  while the Upper Bound for  $X_2 = 5.0 + 0.2 * 1.645 = 5.3$ . The confidence bounds overlap, so, the rule of thumb would indicate that the estimates are not significantly different at the 90% level.

However, if we apply the statistical significance test we obtain:

$$Z = \frac{6 - 5}{\sqrt{(0.5)^2 + (0.2)^2}} = 1.857$$

$Z = 1.857 > 1.645$  which means that the difference is significant (at the 90% level).

All statistical testing in ACS data products is based on the 90 percent confidence level. Users should understand that all testing was done using *unrounded* estimates and standard errors, and it may not be possible to replicate test results using the rounded estimates and margins of error as published.

Users completing statistical testing may be interested in using the ACS Statistical Testing Tool. This automated tool allows users to input pairs and groups of estimates for comparison. For more information on the Statistical Testing Tool, visit <https://www.census.gov/programs-surveys/acs/guidance/statistical-testing-tool.html>.

## EXAMPLES OF STANDARD ERROR CALCULATIONS

### Example 1 – Calculating the Standard Error from the Margin of Error

The estimated number of males, never married is 45,477,830 as found on summary table B12001 (Sex by Marital Status for the Population 15 Years and Over) for the United States for the period 2012-2016. The margin of error is 162,764. Recall that:

$$\text{Standard Error} = \text{Margin of Error} / 1.645$$

Calculating the standard error using the margin of error, we have:

$$SE(45,477,830) = \frac{162,764}{1.645} = 98,945$$

### Example 2 – Calculating the Standard Error of a Sum or a Difference

We are interested in the total number of people who have never been married. From Example 1, we know the number of males, never married is 45,477,830. From summary table B12001 we have the number of females, never married is 39,563,948 with a margin of error of 127,291. Therefore, the estimated number of people who have never been married is  $45,477,830 + 39,563,948 = 85,041,778$ .

To calculate the approximate standard error of this sum, we need the standard errors of the two estimates in the sum. We calculated the standard error for the number of males never married in Example 1 as 98,945. The standard error for the number of females never married is calculated using the margin of error:

$$SE(39,563,948) = \frac{127,291}{1.645} = 77,381$$

Using formula (2) for the approximate standard error of a sum or difference we have:

$$SE(85,041,778) = \sqrt{98,945^2 + 77,381^2} = 125,610$$

Caution: This method will underestimate or overestimate the standard error if the two estimates interact in either a positive or a negative way.

To calculate the lower and upper bounds of the 90 percent confidence interval around 85,041,778 using the standard error, simply multiply 125,610 by 1.645, then add and subtract the product from 85,041,778. Thus the 90 percent confidence interval for this estimate is  $[85,041,778 - 1.645(125,610)]$  to  $[85,041,778 + 1.645(125,610)]$  or 84,835,150 to 85,248,406.

### Example 3 – Calculating the Standard Error of a Proportion/Percent

We are interested in the percentage of females who have never been married to the number of people who have never been married for the period 2012-2016. The number of females, never married is 39,563,948 and the number of people who have never been married is 85,041,778. To calculate the approximate standard error of this percent, we need the standard errors of the two estimates in the percent.

From Example 2, we know that the approximate standard error for the number of females never married is 77,381 and the approximate standard error for the number of people never married calculated is 125,610.

The estimate is:

$$\left(\frac{39,563,948}{85,041,778}\right) * 100\% = 46.52\%$$

Therefore, using formula (4) for the approximate standard error of a proportion or percent, we have:

$$SE(46.52\%) = 100\% * \left(\frac{1}{85,041,778} \sqrt{77,381^2 - 0.4652^2 * 125,610^2}\right) = 0.06\%$$

To calculate the lower and upper bounds of the 90 percent confidence interval around 46.52 using the standard error, simply multiply 0.06 by 1.645, then add and subtract the product from 46.52. Thus the 90 percent confidence interval for this estimate is:

$$[46.52 - 1.645(0.06)] \text{ to } [46.52 + 1.645(0.06)], \text{ or } 46.42\% \text{ to } 46.62\%.$$

#### **Example 4 – Calculating the Standard Error of a Ratio**

We are interested in the ratio of the number of unmarried males to the number of unmarried females for the period 2012-2016. From Examples 1 and 2, we know that the estimate for the number of unmarried men is 45,477,830 with a standard error of 98,945, and the estimate for the number of unmarried women is 39,563,948 with a standard error of 77,381.

The estimate of the ratio is:

$$45,477,830 / 39,563,948 = 1.149.$$

Using formula (3) for the approximate standard error of this ratio, we have:

$$SE(1.149) = \frac{1}{39,563,948} \sqrt{98,945^2 + 1.149^2 * 77,381^2} = 0.003$$

The 90 percent margin of error for this estimate would be 0.003 multiplied by 1.645, or about 0.005. The 90 percent lower and upper 90 percent confidence bounds would then be  $[1.149 - 1.645(0.003)]$  to  $[1.149 + 1.645(0.003)]$ , or 1.144 and 1.154

#### **Example 5 – Calculating the Standard Error of a Product**

We are interested in the number of single unit detached owner-occupied housing units for the period 2012-2016. The number of owner-occupied housing units is 74,881,068 with a margin of error of 360,470, as found in subject table S2504 (Physical Housing Characteristics for Occupied Housing Units) for the period 2012-2016, and the percent of single unit detached

owner-occupied housing units (called “1, detached” in the subject table) is 82.4 % (0.824 with a margin of error of 0.1 (0.001)).

Therefore, the number of 1-unit detached owner-occupied housing units is:

$$74,881,068 * 0.824 = 61,702,000.$$

Calculating the standard error for the estimates using the margin of error we have:

$$SE(74,881,068) = \frac{360,470}{1.645} = 219,131$$

and

$$SE(0.824) = \frac{0.001}{1.645} = 0.0006079$$

Using formula (6), the approximate standard error for number of 1-unit detached owner-occupied housing units is:

$$SE(61,702,000) = \sqrt{74,881,068^2 * 0.0006079^2 + 0.824^2 * 219,131^2} = 186,213$$

To calculate the lower and upper bounds of the 90 percent confidence interval around 61,702,000 using the standard error, simply multiply 186,213 by 1.645, then add and subtract the product from 61,702,000. Thus the 90 percent confidence interval for this estimate is [61,702,000 - 1.645(186,213)] to [61,702,000 + 1.645(186,213)] or 61,395,680 to 62,008,320.

## CONTROL OF NONSAMPLING ERROR

As mentioned earlier, sample data are subject to nonsampling error. Nonsampling error can introduce serious bias into the data, increasing the total error dramatically over that which would result purely from sampling. While it is impossible to completely eliminate nonsampling error from a survey operation, the Census Bureau attempts to control the sources of such error during the collection and processing operations. Described below are the primary sources of nonsampling error and the programs instituted to control for this error.<sup>3</sup>

---

<sup>3</sup> The success of these programs is contingent upon how well the instructions were carried out during the survey.

## Coverage Error

It is possible for some sample housing units or persons to be missed entirely by the survey (undercoverage). It is also possible for some sample housing units and persons to be counted more than once (overcoverage). Both undercoverage and overcoverage of persons and housing units can introduce bias into the data. Coverage error can also increase both respondent burden and survey costs.

To avoid coverage error in a survey, the frame must be as complete and accurate as possible. For the ACS, the frame is an address list in each state, the source of which is the Master Address File (MAF). An attempt is made to assign each MAF address to the appropriate geographic codes via an automated procedure using the Census Bureau TIGER (Topologically Integrated Geographic Encoding and Referencing) files. A manual coding operation based in the appropriate regional offices is attempted for addresses that could not be automatically coded.

The MAF was used as the source of addresses for selecting sample housing units and mailing questionnaires. TIGER produced the location maps for CAPI assignments. Sometimes the MAF contains duplicates of addresses. This could occur when there is a slight difference in the address such as 123 Main Street versus 123 Maine Street, and can introduce overcoverage.

In the CATI and CAPI nonresponse follow-up phases, efforts were made to minimize the chances that housing units that were not part of the sample were mistakenly interviewed instead of units in sample. If a CATI interviewer called a mail nonresponse case and was not able to reach the exact address, no interview was conducted and the case became eligible for CAPI. During the CAPI follow-up, the interviewer had to locate the exact address for each sample housing unit. If the interviewer could not locate the exact sample unit in a multi-unit structure, or found a different number of units than expected, the interviewers were instructed to list the units in the building and follow a specific procedure to select a replacement sample unit. Person overcoverage can occur when an individual is included as a member of a housing unit but does not meet ACS residency rules.

Coverage rates give a measure of undercoverage or overcoverage of persons or housing units in a given geographic area. Rates below 100 percent indicate undercoverage, while rates above 100 percent indicate overcoverage. Coverage rates are released concurrent with the release of estimates on American FactFinder in the B98 series of detailed tables (Table IDs B98011, B98012, B98013, and B980014). Coverage rate definitions and coverage rates for total population for nation and states are also available in the Sample Size and Data Quality Section of the ACS website, at <https://www.census.gov/acs/www/methodology/sample-size-and-data-quality/>.

## Nonresponse Error

Survey nonresponse is a well-known source of nonsampling error. There are two types of nonresponse error – unit nonresponse and item nonresponse. Nonresponse errors affect survey estimates to varying levels depending on amount of nonresponse and the extent to which the characteristics of nonrespondents differ from those of respondents. The exact amount of nonresponse error or bias on an estimate is almost never known. Therefore, survey researchers generally rely on proxy measures, such as the nonresponse rate, to indicate the potential for nonresponse error.

### Unit Nonresponse

Unit nonresponse is the failure to obtain data from housing units in the sample. Unit nonresponse may occur because households are unwilling or unable to participate, or because an interviewer is unable to make contact with a housing unit. Unit nonresponse is problematic when there are systematic or variable differences in the characteristics of interviewed and non-interviewed housing units. Nonresponse bias is introduced into an estimate when differences are systematic; the nonresponse error of an estimate evolves from variable differences between interviewed and non-interviewed households.

The ACS made every effort to minimize unit nonresponse, and thus, the potential for nonresponse error. First, the ACS used a combination of mail, CATI, and CAPI data collection modes to maximize response. The mail phase included a series of three to four mailings to encourage housing units to return the questionnaire. Subsequently, mail nonrespondents (for which phone numbers are available) were contacted by CATI for an interview. Finally, a subsample of the mail and telephone nonrespondents were contacted by personal visit to attempt an interview. Combined, these three efforts resulted in a very high overall response rate for the ACS.

ACS response rates measure the percent of units with a completed interview. The higher the response rate (and, consequently, the lower the nonresponse rate), the lower the chance that estimates are affected by nonresponse bias. Response and nonresponse rates, as well as rates for specific types of nonresponse, are released concurrent with the release of estimates on American FactFinder in the B98 series of detailed tables (Table IDs B98021 and B98022). Unit response rate definitions and unit response rates by type for the nation and states are also available in the Sample Size and Data Quality Section of the ACS website, at <https://www.census.gov/acs/www/methodology/sample-size-and-data-quality/>.

### Item Nonresponse

Nonresponse to particular questions on the survey can introduce error or bias into the data, as the unknown characteristics of nonrespondents may differ from those of respondents. As a

result, any imputation procedure using respondent data may not completely reflect difference either at the elemental level (individual person or housing unit) or on average.

Some protection against the introduction of large errors or biases is afforded by minimizing nonresponse. In the ACS, item nonresponse for the CATI and CAPI operations was minimized by requiring that the automated instrument receive a response to each question before the next question could be asked. Questionnaires returned by mail were reviewed by computer for content omissions and population coverage and edited for completeness and acceptability. If necessary, a telephone follow-up was made to obtain missing information. Potential coverage errors were included in this follow-up.

Allocation tables provide the weighted estimate of persons or housing units for which a value was imputed, as well as the total estimate of persons or housing units that were eligible to answer the question. The smaller the number of imputed responses, the lower the chance that the item nonresponse is contributing a bias to the estimates. Allocation tables are released concurrent with the release of estimates on American Factfinder in the B99 series of detailed tables with the overall allocation rates across all person and housing unit characteristics in the B98 series of detailed tables (Table IDs B98031 and B98032). Allocation rate definitions and allocation rates by characteristic at the nation, and states are also available in the Sample Size and Data Quality Section of the ACS website, at <https://www.census.gov/acs/www/methodology/sample-size-and-data-quality/>.

## Measurement and Processing Error

Measurement error can arise if the person completing the questionnaire or responding an interviewer's questions responds incorrectly. However, to mitigate this risk, the phrasing survey questions underwent cognitive testing and households were provided detailed instructions on how to complete the questionnaire.

Processing error can be introduced in numerous areas during data collection and capture, including during interviews, during data processing and during content editing.

### Interviewer monitoring

An interviewer could introduce error by:

1. Misinterpreting or otherwise incorrectly entering information given by a respondent.
2. Failing to collect some of the information for a person or household.
3. Collecting data for households that were not designated as part of the sample.

To control for these problems, the work of interviewers was monitored carefully. Field staff was prepared for their tasks by using specially developed training packages that included

hands-on experience in using survey materials. A sample of the households interviewed by CAPI interviewers was also reinterviewed to control for the possibility that interviewers may have fabricated data.

### **Processing Error**

The many phases involved in processing the survey data represent potential sources for the introduction of nonsampling error. The processing of the survey questionnaires includes the keying of data from completed questionnaires, automated clerical review, follow-up by telephone, manual coding of write-in responses, and automated data processing. The various field, coding and computer operations undergo a number of quality control checks to insure their accurate application.

### **Content Editing**

After data collection was completed, any remaining incomplete or inconsistent information was imputed during the final content edit of the collected data. Imputations, or computer assignments of acceptable codes in place of unacceptable entries or blanks, were most often needed either when an entry for a given item was missing or when information reported for a person or housing unit was inconsistent with other information for the same person or housing unit. As in other surveys and previous censuses, unacceptable entries were to allocated entries for persons or housing units with similar characteristics. Imputing acceptable values in place of blanks or unacceptable entries enhances the usefulness of the data.

## ISSUES WITH APPROXIMATING THE STANDARD ERROR OF LINEAR COMBINATIONS OF MULTIPLE ESTIMATES

Several examples are provided here to demonstrate how different the approximated standard errors of sums can be compared to those derived and published with ACS microdata. These examples use estimates from the 2005-2009 ACS 5-year data products.<sup>4</sup>

### Example A

With the release of the 5-year data, detailed tables down to tract and block group will be available. At these geographic levels, many estimates may be zero. As mentioned in the ‘Calculations of Standard Errors’ section, a special procedure is used to estimate the MOE when an estimate is zero. For a given geographic level, the MOEs will be identical for zero estimates. When summing estimates which include many zero estimates, the standard error and MOE in general will become unnaturally inflated. Therefore, users are advised to sum only one of the MOEs from all of the zero estimates.

Suppose we wish to estimate the total number of people whose first reported ancestry was ‘Subsaharan African’ in Rutland County, Vermont.

Table A: 2005-2009 Ancestry Categories from Table B04001: First Ancestry Reported

First Ancestry Reported Category	Estimate	MOE
Subsaharan African:	48	43
Cape Verdean	9	15
Ethiopian	0	93
Ghanaian	0	93
Kenyan	0	93
Liberian	0	93
Nigerian	0	93
Senegalese	0	93
Sierra Leonean	0	93
Somalian	0	93
South African	10	16
Sudanese	0	93
Ugandan	0	93
Zimbabwean	0	93

<sup>4</sup> Due to differences in the definition, in rare instances summing PUMA estimates within a state may not equal the state estimate.

First Ancestry Reported Category	Estimate	MOE
African	20	33
Other Subsaharan African	9	16

Source: 2005-2009 American Community Survey 5-year Estimates

To estimate the total number of people, we add up all of the categories.

$$\text{Total Number of People} = 9 + 0 + \dots + 0 + 10 + 0 + \dots + 20 + 9 = 48$$

To approximate the standard error using all of the MOEs we obtain:

$$\text{SE}(\text{total}) = \sqrt{\left(\frac{15}{1.645}\right)^2 + \left(\frac{93}{1.645}\right)^2 + \dots + \left(\frac{16}{1.645}\right)^2 + \dots + \left(\frac{33}{1.645}\right)^2 + \left(\frac{16}{1.645}\right)^2} = 189.3$$

Using only one of the MOEs from the zero estimates, we obtain:

$$\text{SE}(\text{total}) = \sqrt{\left(\frac{15}{1.645}\right)^2 + \left(\frac{93}{1.645}\right)^2 + \left(\frac{16}{1.645}\right)^2 + \left(\frac{33}{1.645}\right)^2 + \left(\frac{16}{1.645}\right)^2} = 62.2$$

From the table, we know that the actual MOE is 43, giving a standard error of  $43 / 1.645 = 26.1$ . The first method is roughly seven times larger than the actual standard error, while the second method is roughly 2.4 times larger.

Leaving out all of the MOEs from zero estimates we obtain:

$$\text{SE}(\text{total}) = \sqrt{\left(\frac{15}{1.645}\right)^2 + \left(\frac{16}{1.645}\right)^2 + \left(\frac{33}{1.645}\right)^2 + \left(\frac{16}{1.645}\right)^2} = 26.0$$

In this case, it is very close to the actual SE. This is not always the case, as can be seen in the examples below.

## Example B

Suppose we wish to estimate the total number of males with income below the poverty level in the past 12 months using both state and PUMA level estimates for the state of Wyoming.

Part of the detailed table B17001<sup>5</sup> is displayed below with estimates and their margins of error in parentheses.

Table B: 2005-2009 ACS estimates of Males with Income Below Poverty from table B17001: Poverty Status in the Past 12 Months by Sex by Age

Characteristic	Wyoming	PUMA 00100	PUMA 00200	PUMA 00300	PUMA 00400
Male	21,769 (1,480)	4,496 (713)	5,891 (622)	4,706 (665)	6,676 (742)
Under 5 Years	3,064 (422)	550 (236)	882 (222)	746 (196)	886 (237)
5 Years Old	348 (106)	113 (65)	89 (57)	82 (55)	64 (44)
6 to 11 Years Old	2,424 (421)	737 (272)	488 (157)	562 (163)	637 (196)
12 to 14 Years Old	1,281 (282)	419 (157)	406 (141)	229 (106)	227 (111)
15 Years Old	391 (128)	51 (37)	167 (101)	132 (64)	41 (38)
16 and 17 Years Old	779 (258)	309 (197)	220 (91)	112 (72)	138 (112)
18 to 24 Years Old	4,504 (581)	488 (192)	843 (224)	521 (343)	2,652 (481)
25 to 34 Years Old	2,289 (366)	516 (231)	566 (158)	542 (178)	665 (207)
35 to 44 Years Old	2,003 (311)	441 (122)	535 (160)	492 (148)	535 (169)
45 to 54 Years Old	1,719 (264)	326 (131)	620 (181)	475 (136)	298 (113)
55 to 64 Years Old	1,766 (323)	343 (139)	653 (180)	420 (135)	350 (125)
65 to 74 Years Old	628 (142)	109 (69)	207 (77)	217 (72)	95 (55)
75 Years and Older	573 (147)	94 (53)	215 (86)	176 (72)	88 (62)

Source: 2005-2009 American Community Survey 5-year Estimates

The first way is to sum the thirteen age groups for Wyoming:

$$\text{Estimate(Male)} = 3,064 + 348 + \dots + 573 = 21,769.$$

The first approximation for the standard error in this case gives us:

$$\text{SE(Male)} = \sqrt{\left(\frac{422}{1.645}\right)^2 + \left(\frac{106}{1.645}\right)^2 + \dots + \left(\frac{147}{1.645}\right)^2} = 696.6$$

A second way is to sum the four PUMA estimates for Male to obtain:

---

<sup>5</sup> Table C17001 is used in this example for the 2009 1-year Accuracy documents. C17001 is not published for the 2005-2009 5-year data.

Estimate(Male) = 4,496 + 5,891 + 4,706 + 6,676 = 21,769 as before.

The second approximation for the standard error yields:

$$SE(\text{Male}) = \sqrt{\left(\frac{713}{1.645}\right)^2 + \left(\frac{622}{1.645}\right)^2 + \left(\frac{665}{1.645}\right)^2 + \left(\frac{742}{1.645}\right)^2} = 835.3$$

Finally, we can sum up all thirteen age groups for all four PUMAs to obtain an estimate based on a total of 52 estimates:

$$\text{Estimate(Male)} = 550 + 113 + \dots + 88 = 21,769$$

And the third approximated standard error is

$$SE(\text{Male}) = \sqrt{\left(\frac{422}{1.645}\right)^2 + \left(\frac{106}{1.645}\right)^2 + \dots + \left(\frac{62}{1.645}\right)^2} = 721.9$$

However, we do know that the standard error using the published MOE is 1,480 / 1.645 = 899.7. In this instance, all of the approximations under-estimate the published standard error and should be used with caution.

### Example C

Suppose we wish to estimate the total number of males at the national level using age and citizenship status. The relevant data from table B05003 is displayed in table C below.

Table C: 2005-2009 ACS estimates of males from B05003: Sex by Age by Citizenship Status

Characteristic	Estimate	MOE
Male	148,535,646	6,574
Under 18 Years	37,971,739	6,285
Native	36,469,916	10,786
Foreign Born	1,501,823	11,083
Naturalized U.S. Citizen	282,744	4,284
Not a U.S. Citizen	1,219,079	10,388
18 Years and Older	110,563,907	6,908
Native	93,306,609	57,285
Foreign Born	17,257,298	52,916

Characteristic	Estimate	MOE
Naturalized U.S. Citizen	7,114,681	20,147
Not a U.S. Citizen	10,142,617	53,041

Source: 2005-2009 American Community Survey 5-year Estimates

The estimate and its MOE are actually published. However, if they were not available in the tables, one way of obtaining them would be to add together the number of males under 18 and over 18 to get:

$$\text{Estimate(Male)} = 37,971,739 + 110,563,907 = 148,535,646$$

And the first approximated standard error is

$$\text{SE(Male)} = \text{SE}(148,535,646) = \sqrt{\left(\frac{6,285}{1.645}\right)^2 + \left(\frac{6,908}{1.645}\right)^2} = 5,677.4$$

Another way would be to add up the estimates for the three subcategories (Native, and the two subcategories for Foreign Born: Naturalized U.S. Citizen, and Not a U.S. Citizen), for males under and over 18 years of age. From these six estimates we obtain:

$$\begin{aligned} \text{Estimate(Male)} &= 36,469,916 + 282,744 + 1,219,079 \\ &\quad + 93,306,609 + 7,114,681 + 10,142,617 \\ &= 148,535,646 \end{aligned}$$

With a second approximated standard error of:

$$\begin{aligned} \text{SE(Male)} &= \text{SE}(148,535,646) \\ &= \sqrt{\left(\frac{10,786}{1.645}\right)^2 + \left(\frac{4,284}{1.645}\right)^2 + \left(\frac{10,388}{1.645}\right)^2 + \left(\frac{57,285}{1.645}\right)^2 + \left(\frac{20,147}{1.645}\right)^2 + \left(\frac{53,041}{1.645}\right)^2} \\ &= 49,920.0 \end{aligned}$$

We do know that the standard error using the published margin of error is  $6,574 / 1.645 = 3,996.4$ . With a quick glance, we can see that the ratio of the standard error of the first method to the published-based standard error yields 1.42; an over-estimate of roughly 42%, whereas the second method yields a ratio of 12.49 or an over-estimate of 1,149%. This is an example of what could happen to the approximate SE when the sum involves a controlled estimate. In this case, it is sex by age.

## Example D

Suppose we are interested in the total number of people aged 65 or older and its standard error. Table D shows some of the estimates for the national level from table B01001 (the estimates in gray were derived for the purpose of this example only).

Table D: Some Estimates from AFF Table B01001: Sex by Age for 2005-2009

Age Category	Estimate, Male	MOE, Male	Estimate, Female	MOE, Female	Total	Approximated MOE, Total
65 and 66 years old	2,248,426	8,047	2,532,831	9,662	4,781,257	12,574
67 to 69 years old	2,834,475	8,953	3,277,067	8,760	6,111,542	12,526
70 to 74 years old	3,924,928	8,937	4,778,305	10,517	8,703,233	13,801
75 to 79 years old	3,178,944	9,162	4,293,987	11,355	7,472,931	14,590
80 to 84 years old	2,226,817	6,799	3,551,245	9,898	5,778,062	12,008
85 years and older	1,613,740	7,058	3,540,105	10,920	5,153,845	13,002
Total	16,027,330	20,119	21,973,540	25,037	38,000,870	32,119

Source: 2005-2009 American Community Survey 5-Year Estimates

To begin we find the total number of people aged 65 and over by simply adding the totals for males and females to get  $16,027,330 + 21,973,540 = 38,000,870$ . One way we could use is summing males and female for each age category and then using their MOEs to approximate the standard error for the total number of people over 65.

$$\text{MOE}(\text{Age 65 and 66}) = \text{MOE}(4,781,257) = \sqrt{8,047^2 * 9,662^2} = 12,574$$

$$\text{MOE}(\text{Age 67 to 69}) = \text{MOE}(6,111,542) = \sqrt{8,953^2 * 8,760^2} = 12,526$$

... etc. ...

Now, we calculate for the number of people aged 65 or older to be 38,000,870 using the six derived estimates and approximate the standard error:

$$\begin{aligned} \text{SE}(38,000,870) &= \sqrt{\left(\frac{7,644}{1.645}\right)^2 + \left(\frac{7,614}{1.645}\right)^2 + \left(\frac{8,390}{1.645}\right)^2 + \left(\frac{8,870}{1.645}\right)^2 + \left(\frac{7,300}{1.645}\right)^2 + \left(\frac{7,904}{1.645}\right)^2} \\ &= 32,119 \end{aligned}$$

For this example the estimate and its MOE are published in table B09017. The total number of people aged 65 or older is 38,000,870 with a margin of error of 4,944. Therefore the published-based standard error is:

$$\text{SE}(38,000,870) = \frac{4,944}{1.645} = 3,005$$

The approximated standard error, using six derived age group estimates, yields an approximated standard error roughly 10.7 times larger than the published-based standard error.

As a note, there are two additional ways to approximate the standard error of people aged 65 and over in addition to the way used above. The first is to find the published MOEs for the males age 65 and older and of females aged 65 and older separately and then combine to find the approximate standard error for the total. The second is to use all twelve of the published estimates together, that is, all estimates from the male age categories and female age categories, to create the SE for people aged 65 and older. However, in this particular example, the results from all three ways are the same. So no matter which way you use, you will obtain the same approximation for the SE. This is different from the results seen in example B.

### Example E

For an alternative to approximating the standard error for people 65 years and older seen in part D, we could find the estimate and its SE by summing all of the estimate for the ages less than 65 years old and subtracting them from the estimate for the total population. Due to the large number of estimates, Table E does not show all of the age groups. In addition, the estimates in part of the table shaded gray were derived for the purposes of this example only and cannot be found in base table B01001.

Table E: Some Estimates from AFF Table B01001: Sex by Age for 2005-2009:

Age Category	Estimate, Male	MOE, Male	Estimate, Female	MOE, Female	Total	Estimated MOE, Total
Total Population	148,535,646	6,574	152,925,887	6,584	301,461,533	9,304
Under 5 years	10,663,983	3,725	10,196,361	3,557	20,860,344	5,151
5 to 9 years old	10,137,130	15,577	9,726,229	16,323	19,863,359	22,563
10 to 14 years old	10,567,932	16,183	10,022,963	17,199	20,590,895	23,616
...	...	...	...	...		
62 to 64 years old	3,888,274	11,186	4,257,076	11,970	8,145,350	16,383
Total for Age 0 to 64 years old	132,508,316	48,688	130,952,347	49,105	263,460,663	69,151
Total for Age 65 years and older	16,027,330	49,130	21,973,540	49,544	38,000,870	69,774

Source: 2005-2009 American Community Survey 5-year Estimates

An estimate for the number of people age 65 and older is equal to the total population minus the population between the ages of zero and 64 years old:

Number of people aged 65 and older:  $301,461,533 - 263,460,663 = 38,000,870$ .

The way to approximate the SE is the same as in part D. First we will sum male and female estimates across each age category and then approximate the MOEs. We will use that information to approximate the standard error for our estimate of interest:

$$\text{MOE}(\text{Total Population}) = \text{MOE}(301,461,533) = \sqrt{6,574^2 * 6,584^2} = 9,304$$

$$\text{MOE}(\text{Under 5 years}) = \text{MOE}(20,860,344) = \sqrt{3,725^2 * 3,557^2} = 5,151$$

... etc. ...

And the SE for the total number of people aged 65 and older is:

$$\begin{aligned} \text{SE}(\text{Age 65 and older}) &= \text{SE}(38,000,870) \\ &= \sqrt{\left(\frac{9,304}{1.645}\right)^2 + \left(\frac{5,151}{1.645}\right)^2 + \left(\frac{22,563}{1.645}\right)^2 + \left(\frac{23,616}{1.645}\right)^2 + \dots + \left(\frac{16,383}{1.645}\right)^2} \\ &= 42,416 \end{aligned}$$

Again, as in Example D, the estimate and its MOE are published in B09017. The total number of people aged 65 or older is 38,000,870 with a margin of error of 4,944. Therefore the standard error is:

$$\text{SE}(38,000,870) = 4,944 / 1.645 = 3,005.$$

The approximated standard error using the seventeen derived age group estimates yields a standard error roughly 14.1 times larger than the actual SE.

## Additional Resources

Data users can mitigate the problems shown in examples A through E to some extent by utilizing a collapsed version of a detailed table (if it is available) which will reduce the number of estimates used in the approximation. These issues may also be avoided by creating estimates and SEs using the Public Use Microdata Sample (PUMS), the Variance Replicate Tables, or by requesting a custom tabulation, a fee-based service offered under certain conditions by the Census Bureau.

For more information on the Variance Replicate Tables, visit: <https://census.gov/programs-surveys/acs/data/variance-tables.html>

For the Variance Replicate Documentation (including table list and summary level list), visit: <https://census.gov/programs-surveys/acs/technical-documentation/variance-tables.html>

Finally, for more information regarding custom tabulations, visit: <https://www.census.gov/programs-surveys/acs/data/custom-tables.html>.