MEASUREMENT OF GEOGRAPHIC AREA

Chapter I. Geography and Area

THE field of geography is concerned with analyzing and interpreting the spatial interrelations of describable phenomena capable of being definitely localized on the earth. Without location, definable by essentially precise latitudinal and longitudinal designation, such phenomena lack a primary requisite for geographic analysis. Localizable and describable phenomena of the earth, whether a microscopic spot, an island, a continent or a region, have at least two dimensions and therefore possess area.[1] A written description of a region by latitude and longitude, besides being an exceedingly laborious task, fails to convey a meaningful mental image. Maps have solved the initial problem of picturing geographic regions, but even maps do not provide the means for a precise areal comparison between regions or between one region and the total area of the earth. Since such areal comparisons are considered essential to geographic analysis, strong impetus has been given to the development of methods for measuring geographic area. Yet in the development of the techniques of area measurement, the first and the majority of the basic contributions were made by astronomers, mathematicians, geodesists, cartographers, engineers, and merchants. Participation in this work by geographers has been delayed although their contributions have been substantial.

In analyzing the spatial interrelations of localizable and describable earth phenomena the geographer is concerned with the quantity of a given phenomenon in terms of definite units of area. To render such concepts comparable in areal terms the device of density per unit of area was developed.[2] Areal densities of localizable and describable phenomena were then put to further use by geographers through the development of various techniques of statistical cartography, namely, isopleths,[3] cartograms and choropleths,[4] and dasymetric adjustments.[5] These techniques use the concept of density per unit area and seek to show the distribution of relevant phenomena for large areas or regions. To serve these techniques for portraying density distribution, geographers in recent years have been actively stimulated to contribute to the work of area measurement.

[1] Hartshorne, Richard, *The Nature of Geography* (Cambridge, Mass.: Association of American Geographers, 1939); and Hettner, Alfred, *Die Geographie, ihre Geschichte, ihr Wesen und ihre Methoden* (Breslau, 1927). These two works supplement each other and provide adequate analyses of the philosophical foundation of geography. They demonstrate how area and the spatial interrelations of describable phenomena within area are basic to geography.

[2] The identity of the person first using density comparisons has not been determined, although an early exponent was the German demographer, J. P. Süssmilch, 1741.

[3] Eckert, Max. *Die Kartenwissenschaft.* (Berlin and Leipzig: W. de Gruyter & Co., 1921–25), Vol. 2; and Jones, W. D. "Ratios and Isopleth Maps in Regional Investigation of Agricultural Land Occupance," *Annals of the Association of American Geographers*, Vol. 20 (1930), 177–195.

[4] Wright, John K. (editor). *Notes on Statistical Mapping.* (New York and Washington: American Geographical Society and Population Association of America, 1938). Mimeographed. 37 pp.

[5] Wright, John K. "A Method of Mapping Densities of Population: With Cape Cod As An Example," *Geographical Review*, Vol. 26 (1936), 103–110.

Chapter II. Geodesy, Cartography, and Area

SEVERAL thousand years of cumulative progress in human understanding were required before accurate measurements could be made of geographic areas stretching beyond the horizon and across seas and oceans. The measurement of such areas was predicated on accurate maps, on the development of mathematical principles, and on the invention of precision instruments. Attention is directed to the history of the measurement of geographic area with the full realization that this development is but one facet of the evolution of geodesy and cartography.

Contemporary primitive artifacts suggest that a form of map making was practiced for centuries before the advent of written records. These maps undoubtedly were in the form of pictorial sketches for relatively small areas. Modern cartography required the development of astronomical knowledge, for accurate maps could not be constructed or envisioned as parts of a world system without an understanding of the shape and size of the earth.

Astronomical behavior was carefully recorded by priests and astrologers in Egypt, Babylonia, and China centuries before the beginning of our era. There is little evidence, however, that they formulated any definite ideas concerning the figure of the earth. Unquestionably there was speculation concerning the apparent movement of the sun and stars from east to west, the changing angular position of the sun and stars with travel north or south, the arc of the eclipse of the moon, and the phenomenon of ships, mountains and other objects gradually receding beyond the horizon with increasing distance. Probably the first written records depicting the sphericity of the earth—attributable to Greek scholars, notably Pythagoras (582–507 B. C.)—were based on a long evolutionary process. Aristotle (384–322 B. C.) in the second book of his treatise De Caelo suggests this development and provides the best summary of Greek opinion then current concerning the sphericity of the earth.

Its shape must necessarily be spherical. For every portion of earth has weight until it reaches the centre, and the jostling of parts greater and smaller would bring about not a waved surface, but rather compression and convergence of part and part until the centre is reached. The process should be conceived by supposing the earth to come into being in the way that some of the natural philosophers describe. Only they attribute the downward movement to constraint, and it is better to keep to the truth and say that the reason of this motion is that a thing which possesses weight is naturally endowed with a centripetal movement. When the mixture, then, was merely potential, the things that were separated off moved similarly from every side towards the centre. Whether the parts which came together at the centre were distributed at the extremities evenly, or in some other way, makes no difference. If, on the one hand, there were a similar movement from each quarter of the extremity to the single centre, it is obvious that the resulting mass would be similar on every side. For if an equal amount is added on every side the extremity of the mass will be everywhere equidistant from its centre, i. e. the figure will be spherical. . . .

If the earth was generated, then, it must have been formed in this way, and so clearly its generation was spherical; and if it is ungenerated and has remained so always, its character must be that which the initial generation, if it had occurred, would have given it. But the spherical shape, necessitated by this argument, follows also from the fact that the motions of heavy bodies always make equal angles, and are not parallel. This would be the natural form of movement towards what is naturally spherical. Either then the earth is spherical or it is at least naturally spherical. And it is right to call anything that which nature intends it to be, and which belongs to it, rather than that which it is by constraint and contrary to nature. The evidence of the senses further corroborates this. How else would eclipses of the moon show segments shaped as we see them? As it is, the shapes which the moon itself each month shows are of every kind—straight, gibbous, and concave—but in eclipses the outline is always curved: and, since it is the interposition of the earth that makes the eclipse, the form of this line will be caused by the form of the earth's surface, which is therefore spherical. Again, our observations of the stars make it evident, not only that the earth is circular, but also that it is a circle of no great size. For quite a small change of position to south or north causes a manifest alteration of the horizon. There is much change, I mean, in the stars which are overhead, and the stars seen are different, as one moves northward or southward. Indeed there are some stars seen in Egypt and in the neighborhood of Cyprus which are not seen in the northerly regions; and stars, which in the north are never beyond the range of observation, in those regions rise and set. All of which goes to show not only that the earth is circular in shape, but also that it is a sphere of no great size: for otherwise the effect of so slight a change of place would not be so quickly apparent. . . . those mathematicians who try to calculate the size of the earth's circumference arrive at the figure 400,000 stades. This indicates not only that the earth's mass is spherical in shape, but also that as compared with the stars it is not of great size.[6]

[6] Stocks, J. L. (*Aristoteles*) *De Caelo*, (Oxford: Clarendon Press, 1922), 297*a*, 297*b*, and 298*a*. This circumference of 400,000 stadia, in round numbers equals 46,000 miles on the basis of 606.9 English feet for each stadium. The earth's equatorial and polar circumferences are given as 24,901 and 24,860 miles respectively and show this Greek dimension to be grossly in error.

The figure quoted by Aristotle was probably little more than a shrewd guess. Not until the work of the Alexandrian Eratosthenes (276–196 B. C.) is it certain that actual measurement formed the basis for computing the circumference of the earth. The oft repeated, well illustrated story of Eratosthenes' famous determinations, based on simultaneous observations of the angle of the sun at Aswan and Alexandria resulted in the remarkably accurate conclusion of an earth circumference of 250,000 stadia, or approximately 28,000 miles. This work was followed by the calculations of Posidonius (130–50 B. C.), based on the distance from Rhodes to Alexandria and the height of the star Canopus above the horizon. The inaccuracy of the result, 18,000 miles, is largely attributable to the fact that the distance between the two points was not accurately known and no allowance was made for the refraction of light by the earth's atmosphere. Posidonius' small figure for the circumference of the earth was given authority by the maps of Ptolemy (90–168 A. D.), which maps unquestionably influenced the optimism of Columbus in his westward journey of discovery.[7]

After the overthrow of the Alexandrian School, which had flourished under Roman protection, a dark age apparently lacking in much speculation concerning the figure of the earth prevailed until the Empire of the Arabian Caliphs. The Arabians translated the works of the Greek mathematicians in the eighth and ninth century and made determinations of the length of two degrees of latitude in the plains of Mesopotamia. It is known that they possessed fairly good astrolabes and measured surface distances with cords and rods. Their measurements, conducted by two independent surveying parties, were 56 miles of 4,000 cubits for a one degree arc and 56⅔ miles for the other. Unfortunately the accuracy of these measurements cannot be checked since the length of a cubit is not known, but in view of the methods employed they are considered to be accurate. With the decline of Arabian interest, the shape and size of the earth ceased to be an object of more than isolated speculation for seven hundred years.

Aside from this independent Arabian development, unappreciated by the occidental world until recent times, no recorded work was done to determine the figure of the earth from the decline of the Alexandrian School until after the renaissance of cartography initiated by the rediscovery and translation of Ptolemy's *"Geographia"* during the first decade of the fifteenth century.[8] Cartographers promptly began to improve their maps by using the continental outlines and latitudinal and longitudinal grid of Ptolemy. The earth was generally regarded as a sphere by the learned. Discovery and exploration by Columbus, de Gama, Magellan, and others provided new geographic data and demonstrated the gross errors of the Ptolemaic maps. New and improved maps were drawn. Interest in place geography became greatly stimulated. The invention of printing and engraving made possible the wide dissemination of many improved maps at low cost. The stage was set for new efforts to determine more accurately the true figure of the earth.

Four centuries of intensive effort to determine the size and shape of the earth were initiated by Fernel, a Frenchman, when he published an account (1528) of his measurement of an arc of latitude. Starting from a point in Paris, he measured the angle of the sun's meridian altitude with a triangular device having one vertical side eight feet in length, another of the same length attached at the bottom end of the first and movable, and a third graduated length connecting the other *two*. He measured northward from the starting point by counting the number of revolutions made by the wheel of his carriage, a distance which he judged to be sufficient. Fernel took a reading of the meridian angle of the sun from this new position, adjusted for the difference in declination caused by the lapse of four days' time, and calculated the length of one degree of latitude to be 56,746 toises, which, considering the means used, compares so favorably with 57,060 toises given by modern observations that a liberal allowance must be made for the good fortune of compensating error.

Since the design for the crude triangular device used by Fernel dated back to before the time of Ptolemy, the modern period of geodetic research really began with the work of the Dutchman Snell who published his results in Leyden, Holland, in 1617. Snell greatly improved the accuracy of measurement by the use of rods and perambulators, far less subject to error than the jerky wheel of a horse-drawn carriage, and by the

[7] Raisz, Erwin, *General Cartography* (New York: McGraw-Hill Book Co., Inc., 1938), 14–18. The statements made by Raisz are paralleled with interesting additional detail by: Airy, George Biddell, "Figure of the Earth," *Encyclopaedia Metropolitana*, V, (London: William Clowes & Sons, 1845), 165 and 174; and Wagner, Hermann, *Lehrbuch der Geographie: Einleitung. Allgemeine Erdkunde*, Erster Band (Hannover: Hahnsche Buchhandlung, 1900), 91–94. Primary dependence for the statements made tracing the development of cartography, except where otherwise noted, has been placed on these three sources, since, as should be apparent, the emphasis of this study is on the measurement of geographic area.

[8] In spite of this extended period of intellectual retrogression and supernaturalism when it was generally believed even among educated clerics that the earth was flat and disc-shaped, the concept of a spherical earth and orderly planetary motion was apparently not lost judging by the Paradiso of Dante's Divine Comedy written during the first two decades of the fourteenth century. A lineal and rhymed translation is provided by Bandini, Albert R., *Dante's Divine Comedy* (San Francisco: The People's Publishing Co., 1928). For further evidence see Sarton, *Introduction to the History of Science*, Vol. I.

substitution of trigonometric calculation for the less accurate measurement of whole distances. The Englishman, Richard Norwood, improved the methods of Snell by the use of chains with a length of 99 feet and by accurately observing road bearings with a circumferentor; Picard, whose work was begun in 1669 in France, added considerable refinement by using a telescope equipped with cross-hairs for the determination of the meridian altitude of stars.

Huygens and Newton introduced a new problem for the geodesist. Prior to the work on centrifugal force which Huygens published in 1673, and which Newton drew upon in his *Principia*, published in 1687, the sphericity of the earth had not been questioned for two or more centuries. The goal of geodetic research had been to establish the exact size of the earth-sphere. Newton reasoned from the theory of centrifugal force that the earth is not rigid; that the shape of the earth must be that which a homogeneous fluid body would assume if it rotated as does the earth. On the basis of the theory of centrifugal force and his principle of gravitation, Newton calculated that the earth must be an oblate spheroid with an ellipticity of $\frac{1}{230}$. Observable evidence of the variation of gravity on the earth's surface, such as the astronomically demonstrable time variations of accurate clocks due to latitudinal change, served to confirm this conclusion. Since the work of Newton, geodetic research has been redirected to establish the shape as well as the size of the earth.

Assuming the earth to be an oblate spheroid, then the length of degree arcs of meridian would increase from the equator to the poles.[9] Measurements made by J. Cassini (between 1684 and 1734), used in conjunction with those of Picard, gave the opposite conclusion and depicted the earth as elongated. To settle the controversy which arose, two French expeditions engaged in measuring the length of degree arcs of meridian in Peru (1735–43) and Lapland (1736–37). The measurements made by these expeditions, though not without considerable error, served to establish the fact that the length of a degree of latitude increases from the equator to the poles and demonstrate the soundness of Newton's theory concerning the oblateness of the earth. During the next one hundred years geodetic knowledge was extended by the work of a number of new expeditions and surveys. The accuracy of measurements taken in the field was greatly improved through the development of precision surveying instruments. The use of telegraphic impulses in transmitting time signals made possible highly accurate determinations of longitude. Careful study of the isostatic effect of mountain ranges and oceans on the behavior of plumb-lines or pendulums, used in surveying to determine the true zenith, contributed to the determination of accurate geodetic dimensions.

This long evolutionary process culminated in the publication in 1841 of the dimensions of the earth ellipsoid calculated by the German astronomer, F. W. Bessel. This work was followed by the reliable dimensions determined by the English geodesist, Alexander Ross Clarke in 1866, which dimensions were refined by telegraphic longitudinal determinations in 1880. The dimensions of Bessel and Clarke though at slight variance from each other, provided the basis for an accurate map grid. Improvements on the work of Bessel and Clarke have been made by other geodesists, notably the American John F. Hayford in 1909.[10] However, the dimensions of Bessel and Clarke, used for a major portion of the world's maps,[11] provided adequately for the basic needs of area measurement. Unfortunately after the lapse of nearly 100 years since the work of Bessel and Clarke, accurate or even fairly reliable maps on a large scale, are available for considerably less than one-half of the earth's land surface (fig. 1).[12]

[9] Meridians form eclipses which possess their maximum curvature at the equator and their minimum curvature at the poles. Since the curvature of the earth is greatest at the equator and least at the poles, a circule corresponding to the curvature of the earth at the equator is smaller and the length of its arc of one degree shorter than a circle corresponding to the curvature of the earth at the poles. The lengths of arcs of latitude of any given angular length increase in a definite progression from the equator to the poles.

For a more complete statement covering this point see Moulton, F. R., *Astronomy* (New York: The Macmillan Co., 1931), 85; or Raisz, *op. cit.*, 76.

[10] Hayford, John F., *Supplementary Investigation in 1909 of the Figure of the Earth and Isostasy* (Washington: U. S. Coast and Geodetic Survey, 1910).

[11] Lambert, W. D., *Effect of Variations in the Assumed Figure of the Earth on the Mapping of a Large Area* (Washington: U. S. Coast and Geodetic Survey Special Publication No. 100, 1924).

[12] Although theoretically it might be desirable for the measurements of John F. Hayford to have been accepted as international standard earth dimensions, the negligible differences which would result from new maps made using these standards would not justify the prohibitive expense and delay which would be involved in their construction. It was with this thought in mind that John F. Hayford recommended to the superintendent of the U. S. Coast and Geodetic Survey that the Clarke spheroid of 1866 as expressed in meters, which had been used for many years by the Survey for its triangulation, be continued in use (Bowie, William, "The Triangulation of North America," *Geographical Journal*, Vol. 72, 348–56). A comparison between the dimensions of the Bessel, Clarke, and Hayford spheroids is instructive:

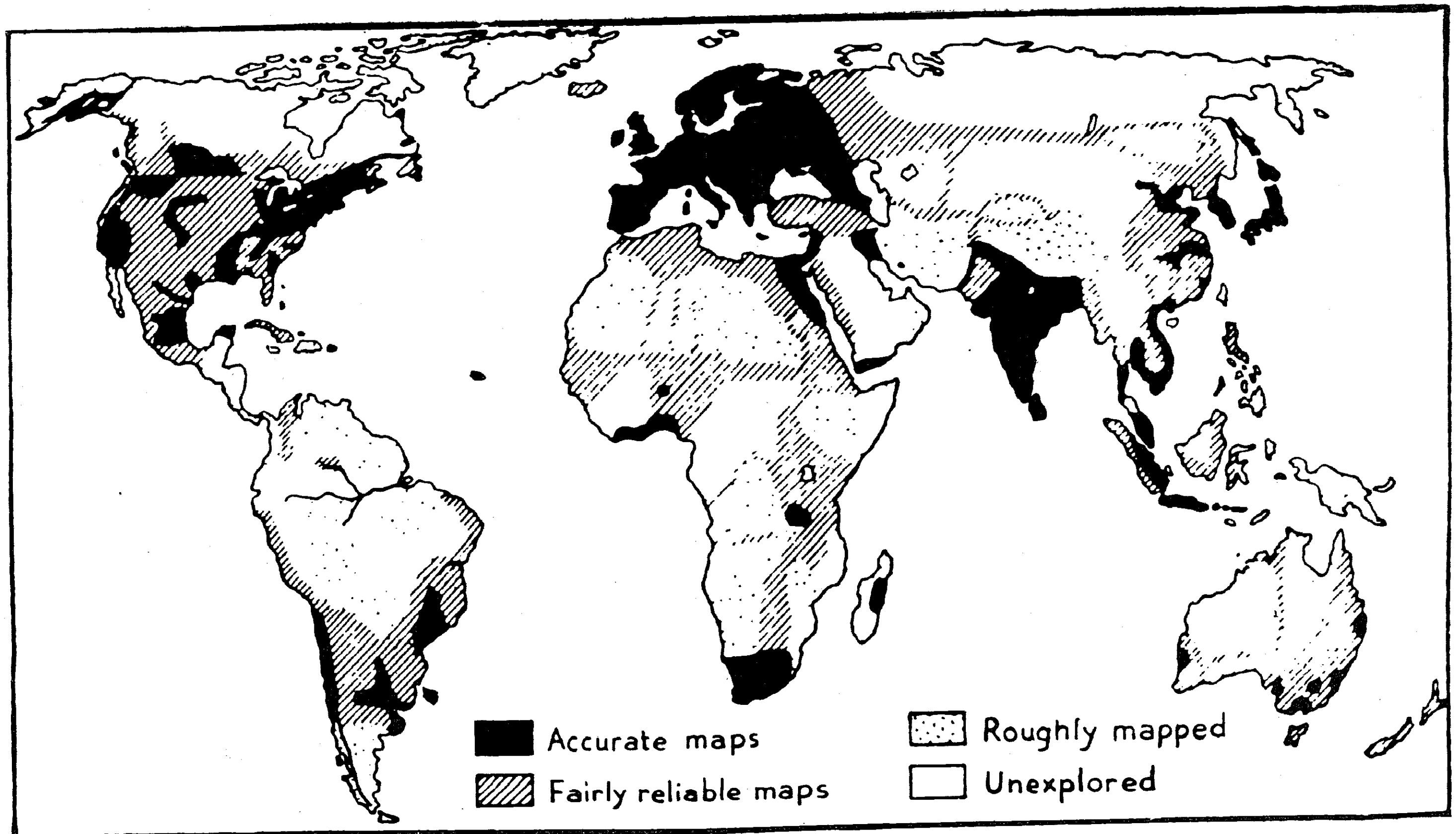
	Date	Equatorial radius in kilometers	Ellipticity
Bessel	1841	6,377.397	1/299.15
Clarke	1866	6,378.207	1/294.98
Hayford	1909	6,378.388	1/297.00

Further to bear out the minor effect which such a change might produce, it is interesting to note a comparison of the total area of Europe computed by I. A. Strielbitsky, *La Superficie de l'Europe* (St. Petersburg, 1882), p. 225, based on the Bessel and Clarke dimensions:

(*a*) Bessel (1841) 9,305,344 square kilometers.

(*b*) Clarke (1866) 9,308,527 square kilometers.

$$\frac{b-a}{b} = 0.034 \text{ percent}$$



State of the topographic mapping of the world (Raisz, E., *General Cartography*, 196).

FIGURE 1.

Reprinted by permission of McGraw Hill Co., Inc., New York, N. Y.

For a larger scale map showing the status of world mapping see "World: Official Topographic Maps," *Geographical Review*, Vol. XXXV, No. 2, April, 1945, facing page 180.

Chapter III. Ancient and Medieval Areas

THE concept of distance appears antecedent to that of area. Primitive contemporary peoples have a well-developed appreciation of distance, although in terms of such indefinite linear measures as the length of a stride or pace, or the average distance covered in a day on foot, on horseback, or in a canoe. This understanding of distance is substantiated by the handmade maps of Eskimos, Marshall Islanders, and those of African and Asiatic primitives.[13] The concept of area, if held by primitive peoples, apparently is in general terms for the length and breadth of tribal holdings based on indefinite boundaries. Perhaps it can be assumed that the concepts of area held by primitive contemporary peoples are similar to those held by primitive peoples before the beginning of written history.

The first historical evidence pertaining to the area concept is found in written references to the size of private landholdings in Egypt.[14] Further evidence is provided for Egypt, and an understanding of the area concept is suggested for Babylonia and China by the discovery of numerous metal and wooden linear measures,[15] the architectural proof of a rudimentary understanding of mathematics, and the remains of stone, baked clay and paper property maps.[16] However, there is no evidence during this period of work done to calculate the areas of large geographic regions and countries. The area concept appears to have been confined to the measurement of small rectangular parcels of land, of little concern to this summary of the measurement of geographic area.

During the Greek and Roman periods it became common to compare continents and countries on the basis of length, breadth and circumference. The writings of Plinius during the first century attest eloquently to this practice. He advanced the scheme of using the sum of the length and breadth of continents as a basis for size comparisons.

> I shall now proceed to compare the dimensions of the various parts of the earth, however great the difficulties which may arise from the discrepancy of the accounts given by various authors: the most convenient method, however, will be that of adding the breadth to the length. Following this mode of reckoning, the dimensions of Europe will be eight thousand two hundred and ninety-four miles; of Africa, to adopt a mean between all the various accounts given by authors, the length is three thousand seven hundred and ninety-four miles, while the breadth, so far as it is inhabited, in no parts exceeds two hundred and fifty miles. But, as Agrippa, including its deserts, makes it from Cyrenaica, a part of it, to the country of the Garamentes, so far as was then known, a further distance of nine hundred and ten miles, the entire length, added together, will make a distance of four thousand six hundred and eight miles. The length of Asia is generally admitted to be six thousand three hundred and seventy-five miles, and the breadth, which ought, properly, to be reckoned from the Aethiopian Sea to Alexandria, near the river Nile, so as to run through Meroe and Syene, is eighteen hundred and seventy-five. It appears then that Europe is greater than Asia, by a little less than one-half of Asia, and greater than Africa by as much again of Africa and one-sixth. If all these sums are added together, it will be clearly seen that Europe is one-third, and a little more than one-eighth part of one-third, Asia one-fourth and one-fourteenth part of one-fourth, and Africa, one-fifth and one-sixtieth part of one-fifth of the whole earth.[17]

Mathematicians of the Alexandrian School immediately pointed out the fallacious reasoning involving in making comparisons such as those drawn by Plinius and others. They condemned the practice of comparing the circumferences of irregular areas, the assumption that the sum of the length and breadth of a continent is representative of its area, and the computation of the area of a country from its dimensions when its shape approximated a geometric figure. An independent development of a similar character resulted among the Arabs. The Arabian geographer Kodama in 922 obtained the area of Iraq by assuming it to possess the shape of a parallelogram. There is no evidence, however, that this or any other area determinations made by the Arabians in any way influenced the occidental world.

[13] Flaherty, R. J., "The Belcher Islands," *Geographical Review*, Vol. 5 (1918), 440; Lyons, H., "The Sailing Charts of the Marshall Islanders," *Geographical Journal*, Vol. 72 (1928), 325–328; and Adler, B. F., "Maps of Primitive Peoples," St. Petersburg (1910), 350 pp., as abstracted by H. de Hutorowicz, *Bulletin of the American Geographical Society*, Vol. 43 (1911), 669–679.

[14] Schmiedeberg, W., "Zur Geschichte der geographischen Flächenmessung bis zur Erfindung des Planimeters," *Zeitschrift der Gesellschaft für Erdkunde*, Nr. 3 and 4 (1906), 152–176 and 233–256. Throughout chapters III, IV and V this basic work by Schmiedeberg has been drawn upon except when otherwise specifically noted. The sources quoted by Schmiedeberg have been examined when available and a number of direct quotations have been made. It was considered appropriate to provide full footnote references to the more important sources cited by Schmiedeberg in order to expedite their examination.

[15] Petrie, Sir Wm. M. F., *Ancient Weights and Measures*, (London, 1926).

[16] Raisz, *op. cit.*, 11–13. For an interesting reference to the means employed in Egypt for measuring fields see: Gelcich, E., "Zur Geschichte der Arealbestimmung eines Landes," *Zeitschrift der Gesellschaft für Erdkunde*, Nr. 123 (1886), 297–298.

[17] *The Natural History of Pliny* (Plinius Secundus, C.), translated by John Bostock and H. T. Riley (London: George Bell and Sons, 1890), Vol. 2, Book VI. Ch. 38, 109–110.

Chapter IV. Developments in Area Measurement

INTEREST in comparing the areas of countries and large irregular shaped geographic regions did not die out during the Middle Ages. Virtual cessation of extended travel and retrogression in geographic and cartographic knowledge must in part explain the apparent absence of written reference to area measurement until the beginning of the seventeenth century. The stage was set for a revival of interest by a century of world voyages of exploration and by the preparation of maps recording the geographical discoveries made on these voyages.

The subject of area measurement was revived by a reconsideration of ancient concepts. Scholars again sought to calculate the areas of irregular shaped countries by the simple process of multiplying their average length by their average breadth. For irregularly shaped areas it was considered adequate to compare estimates of length and breadth and full satisfaction was derived from comparisons of circumference.

One basic step recorded early in this period was the work of Christopher Clauvius (1612) who calculated the surface area of the earth in square stadii and Roman square miles based on the dimensions of Ptolemy, Alphraganus, Fernel and contemporary authors. Clauvius' work was followed by that of others who likewise sought to accurately determine the surface area of the earth. These determinations did not cope with the problem of measuring the area of irregularly shaped countries or geographic regions. They were mathematical calculations based on estimated dimensions rather than on area measurements.

To the English merchant-scholar, Gerard de Malynes (1636) belongs the honor of publishing the first essentially complete tables of the areas for all countries of the world.[18] His tables, comprising one chapter of a large volume of data considered essential to a merchant engaging in foreign trade, by implication bespoke the importance he attached to inter-area comparison. This is borne out by the fact that his areas are given in acres instead of the mathematical measure of square degrees and minutes. As to the method employed by Malynes to obtain his areas, one must assume that he used the laborious "polygonic method" of laying out map surfaces into squares, triangles, parallelograms, and trapezoids, making estimates for the remnant areas along irregular boundaries.[19]

Malynes' areas are, in practically all cases, too small, being 60,000 square kilometers too small for Great Britain and Ireland, and not even one-fourth part of the correct value for the entire earth's surface. These inaccuracies reflect on his work since he gives the circumference of the earth as 21,600 statute miles and from this figure could have calculated a more nearly correct area figure for the surface of the earth.

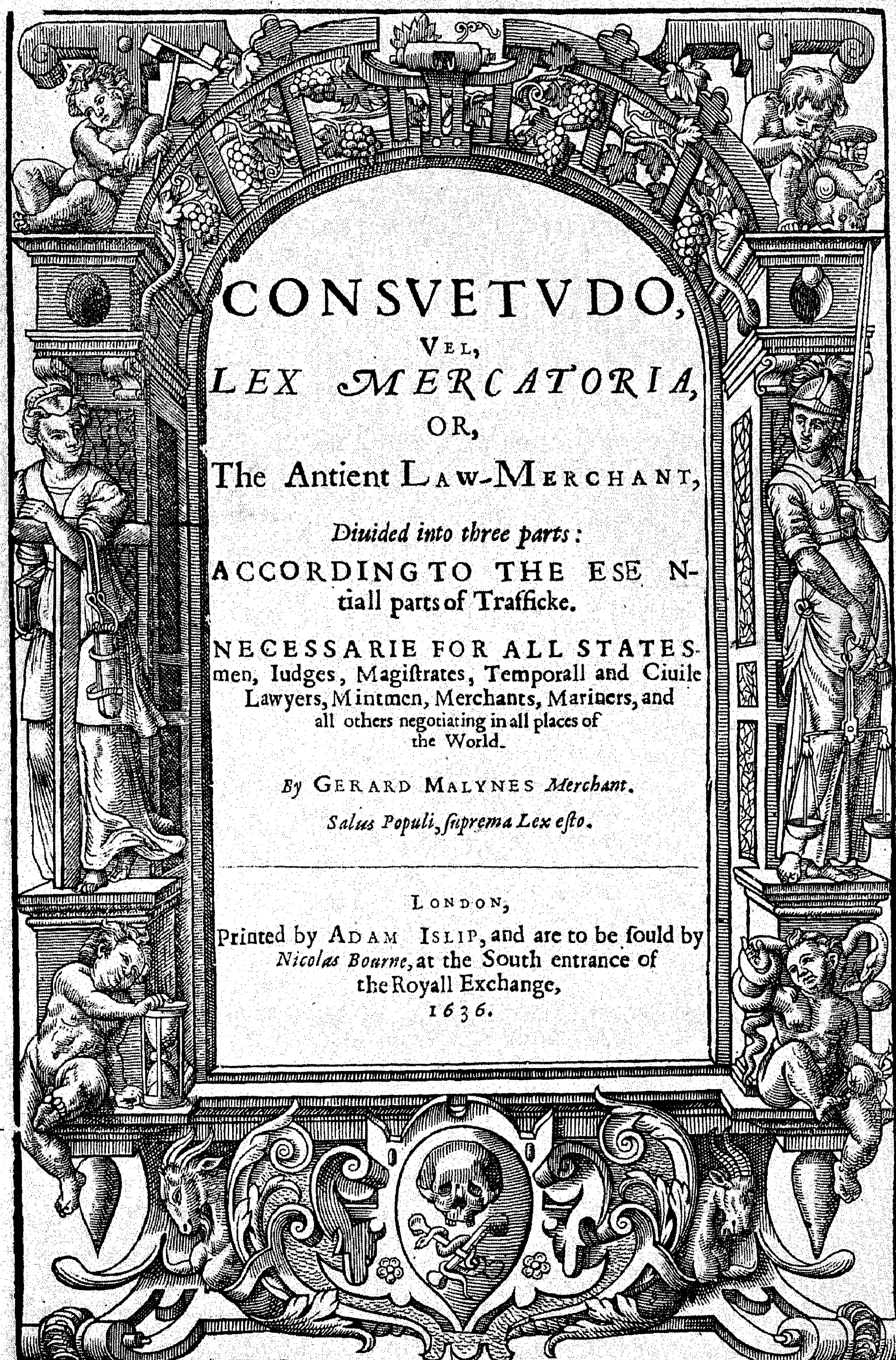
Beginning on the next page is a facsimile of the title page from Malynes' book and all of chapter VI wherein are contained his introductory remarks and area figures.

Indicative of the isolation of those times, the work of Malynes went unnoticed on the Continent. We find Matthias Bernegger, a professor at Strassburg, Germany, as late as 1655, seeking to refute the current practice of comparing the length, breadth and circumference of countries by reference to the work of the Alexandrian mathematicians. He reiterated the conclusion that "the area of a figure is not alone the function of its circumference"; and an Italian, G. B. Riccioli in 1661 characterized Plinius' classical comparison of continents as entirely unmathematical and incorrect. However, some independent progress had been made on the Continent. Riccioli actually asserted the value of inter-area comparison, and he told of determining the area of Italy by smoothing its circumference on the map by means of triangles and parallelograms (the first clear evidence of the use of the polygonic method).

In England the areas of Malynes, after a lag of several decades, were accorded important use by contemporary statisticians. Edward Chamberlaine rounded Malynes' areas and used them in his statistical compilations concerning the population of England. William

[18] Malynes, Gerard de, *Consvetvdo, vel, Lex Mercatoria,* (London: Adam Islip, 1636), 333 pages. Perhaps these tables or similar ones were published earlier, since the first edition of this volume appeared in 1622 and a second in 1629. These earlier editions were not available for inspection.

[19] Geometry was fully understood during the seventeenth century. As far back as the time of Archimedes the areas of circles had been obtained by the process of fitting regular line polygons to their curved portions. It is safe to assume that the areas published by Malynes were obtained through use of the polygonic method. Anyone who has attempted to use this technique to obtain the area of a country with a highly irregular coastline knows the painstaking labor involved.



CONSVETVDO,
VEL,
LEX MERCATORIA,
OR,
The Antient LAW-MERCHANT,

Diuided into three parts:
ACCORDING TO THE ESE N-
tiall parts of Trafficke.

NECESSARIE FOR ALL STATES-
men, Iudges, Magiſtrates, Temporall and Ciuile
Lawyers, Mintmen, Merchants, Mariners, and
all others negotiating in all places of
the World.

By GERARD MALYNES *Merchant.*

Salus Populi, ſuprema Lex eſto.

LONDON,
Printed by ADAM ISLIP, and are to be ſould by
Nicolas Bourne, at the South entrance of
the Royall Exchange,
1636.

price, becaufe they find mony at intereft, at fiue and fix in the hundreth.

This plenty of mony is daily increafed by our merchants trading into Spaine and all others, who doe diuert the Royals of Spaine from vs, becaufe of the inhaunfing of monies beyond the feas, where they haue 25 vpon the hundreth gain, when with vs they make but 10 p cent.

This gain is practifed by exchange, and would otherwife be but imaginary, as fhall be declared hereafter: whereby we fhall finde that the faid Exchange is ftil predominant, and ouer-ruling the monies and commodities.

Chap. VI.

A Geometricall Defcription of the World, efpecially of Evrope, *Meafured by Millions of Acres of Ground, vpon the Map.*

¶ *The Meafure is one Million, or ten hundred thoufand Acres.*

THE Circumference of the roundneffe of the whole Globe of the World, compofed of Water and Earth, is accounted to be 5400 Geometricall miles, or 21600 ordinary miles. But whereas the Miles in all Kingdomes and Countries, and almoft in euery Prouince or Shire do differ; I haue thought conuenient to admit one meafure of one million of Acres of ground, to meafure the whole Globe thereby according to the Map: which is not only intelligible to all men, but all Merchants alfo may haue vfe thereof. For by the number of the millions of Acres, comparing one kingdome vnto another, or one Countrey vnto another Countrey, they may know the bigneffe and fpacioufneffe thereof, which wee haue particularly obferued in Europe, with a diftinction alfo of the dominion of Princes in thefe feueral Countries known by the name, France, Italy, Germany, and others, which many times falleth into confideration vpon fingular occafions.

This Globe of the World is diuided to be two third parts Water or feas, and one third part land: and of this Land there is one third part not inhabited, and the other two third parts are as followeth.

The whole Circumference by the aforefaid meafure is 19, 803, 575000, which is 29 milliars, 803 millions, 575 thoufand Acres, and the milliar is ten hundred millions. So the $\frac{2}{3}$ part water is 19 milliars, 869 millions, and 50000 acres of ground anfwerable; and the other part third is 9 milliars 934 millions, and 525 thoufand Acres. A Milliar is ten hundred Millions.

Hereof deduct $\frac{1}{3}$ part not inhabited, which is 3 milliars, 311 millions, 508000 Acres; fo reft 6 milliars 623 millions, 17000 acres of land inhabited, whereof followes a particular diftribution.

¶ *Firft for Europe or Chriftendome.*

England containeth 29 millions, 568 thoufand Acres.

Scotland containeth 14 millions, 432 thousand Acres.
Ireland containeth 18 millions.
So these three Kingdomes, with all their dominions of lands and Islands adjacent, vnder the Diadem of King *Iames*, contain 62 millions of acres of ground, &c.

England is by this computation, accordingly with the Dominion of Wales, and all Islands thereunto belonging, the thousand part of the whole Globe, or the 22 part of the earth inhabited, or the 333 part of the whole earth: and Scotland may be full the one halfe of this computation, that is, the 444 part of the earth inhabited, or the 666 part of the whole earth; and the Monarchy of Great Brittain and the Kingdome of Ireland is the 480 part of the whole Globe, vnnecessarie fractions in cypher omitted.

The body of the Sun is 166 times bigger than the whole Globe of the World, and so accordingly for the seas and earth as aforesaid.

¶ *The 17 Prouinces of the Low-Countries,*

Containe 10 millions 797 thousand Acres. Whereof
The reconciled Prouinces with Spaine containe 7 millions, 197000 acres.
The vnited Prouinces vnder the States containe 3 millions 599000 acres.
The Kingdome of France diuided into 32 Prouinces, containeth in all 82 millions 879000 acres.
The kingdomes of Spaine, being 8 in number, containe as followeth:

	m.		m.
Castile	25 millions 730	Leon galisia	9 millions 124
Andalusia	2 millions 425	Arragon	13 millions 104
Granado	2 millions 128	Biscay	3 millions 412
Navarre	1 million 458	Portugal	10 millions 154

In all containing 67 Millions, 535 thousand acres.

ITALY.

	m.		m.
Vnder Spaine.		Savoy	1 million 910
Naples	11 millions 704	Piedmont	1 million 160
Lombardy	1 million 640	Toscana, &c.	4 millions 785
Vnder Venice.		Suria and Florence	480
Trevisana	2 millions 584	Marca / Ancona	1 million 412
Verona	0 millions 480	Parma	885
Frioul	1 million 047	Sicilia	3 millions 113
Mantua	0 millions 480	Cypres	1 million 601
Vnder Rome.		Candia	2 millions 060
Liguria	1 million 415	Corsica	1 million 395
Romagnia	1 million 085	Sardegna	4 millions 089
Hetruria	0 millions 540		
Latium	0 millions 480		

Containing in all 44 millions 257 thousand Acres.

GERMANIA.

	m.		m.
Saxonia	3 millions 484	Bavaria	3 millions 249
Misnia	3 millions 249	Helsatia	3 millions 644
Turnigia	1 million 093	Heluetia	12 millions 328
Lusatia	2 millions 572	Basle	0 millions 842

Swebourg

	m.
Swebourg	2 millions 109
Salsbourgh	1 million 063
Trier, Ments, Spiers, Strausbourgh, and Wormes	4 millions 237
Iuliers	348
Cleve	258
Westphalia	2 millions 300
Osnab	358
Silesia	5 millions 706
Bohemia	7 millions 024
Austria	6 millions 121
Moravia	4 millions 114
Pomerania	3 millions 249
Brandenbourg	6 millions 208
Machalbourg	2 millions 107
Franconia	6 millions 361
Tiroll	3 millions 249
Carinthia	1 million 588
Stiria	1 million 779
Palatine Rhene	4 millions 361
Wirtenborgh	1 million 223
Embden	230
Oldenbourgh	449
Liege	548
Cologne	215

Containing in all 95 millions, 646 thousand acres.

Prussia——10 millions 240 thousand Acres.

	m.
Russia	9 millions 607
Volhimia	5 millions 762
Massovia	1 million 916
Livonia	34 millions 115
Poland	19 millions 205

Heretofore named Polonia, containing in all 80 millions 845 m.

DENMARKE.

	m.
Denmarke	10 millions 426
Norway	28 millions 492
Holstein	1 million 065
Ditinars	337

Containing in all 40 millions, 326 thousand acres.

SWETHEN.

Swethen	57 millions 430000 acres.
Finland	7 millions 531000
Gothia	20 millions 936000

Containing in all 85 millions, 897000 acres.

Part of Russia or Muscovia and Situam vnder Europe, 232 millions, 558 thousand acres. So that whole Europe or Christendom doth but contain 802 millions 740000 acres, which is not the 12 part of the whole earth. EVROP.

m.

Hungary, Dalmatia, Transylvania, and all Turky, 385 mill. 367
Muscovia pars 128 millions, 817000 acres.
Tartaria——299 millions, 110000
Persia——385 millions, 367000
Calicut and East India 587 millions, 200000 acres. ASIA.
Africa containeth 1541 millions, 883000 acres. AFRICA
America containeth 1152 millions, 400000 acres.
Nova Hispania 1349 millions, 133000 acres. AMERICA.

H 2 ¶ Sum m

¶ Summa totalis *of the inhabited parts of the World, six Milliars, six hundreth and twenty three Millions, and seuenteen thousand Acres of Land*, Vt supra.

THE vse of this Description, to know the bignesse of one Countrey, compared vnto another countrey, is for example:

England containeth 29 measures and odde; Bohemia containing 7 measures and odde, is the fourth part of England, or thereabouts.

The 17 prouinces of the Low-countries, beeing ten measures and odde, is the third part of England, or thereabout.

One penny an acre, is 100000 pound in England.

The Monarchie of great Brittain being 44 measures and odde, is as big as all Italy and the fiue Islands of Sicilia, Cyprus, Candy, Corsica, and Sardignia, and is also more populous, and so for all other countries: insomuch that England only, containing 29 of those measures, or Millions, 568 thousand acres of ground; if we deduct the 5 Millions, and 568 thousand acres for wilde and waste grounds and highwaies, there wil remain 24 millions of acres, which at one penny an acre, amounteth to 100000 ℔, a matter little sensible.

Here may *Microcosmos*, the little world Man, behold how little the great world is, which is made for him as a tabernacle, where he hath no abiding place, but trauelleth as a Pilgrim toward the celestiall habitation, with a thankfull minde & remembrance of the mercy of God, who hath been mindeful of the son of man, and made him lower than the Angels, to crowne him with glory and worship, *Psal*. 8.

Thus much I thought to demonstrat vnto Christian Merchants touching the world, before we intreat of the commodities, wherby traffique and commerce is by them and other nations maintained; according to the antient Verse,

Currit Mercator ad Indos,—— as appeareth in the next Chapter.

CHAP. VII.

Of the Commodities of all Countries, whereby Commerce is maintained.

Corporum tria sunt genera.

ALbeit that the aforesaid Doctors of the Civill Law haue declared that there be but three kind of bodies of things, namely,

1 *Quod continetur vno spiritu, vt Homo, Lapis, &c.* Which is contained of one spirit, as Man, and a Stone, or such like.

2 *Quod ex pluribus inter se, coherentibus constat, vt edificium nauis, &c.* Which consisteth of many things ioyned together, as a Building, a Ship, &c.

3 *Quod ex distantibus constat, vt corpora vno nomine subiecta, veluti Populus, legio, grex, &c.* Which consisteth of distant things, as many bodies vnder one name, a People, a Legion, a Flocke, and the like. Yet this definition is not compleat concerning the body of Traffique and Commerce, consisting of body, soule, and spirit, namely Commodities,

Petty, John Graunt and others made free use of Malynes and Chamberlaine and in some instances computed their own areas. The areas used by all of these men bore the Malynes stamp of acreage, and in giving total areas wholly too small for England. This ready acceptance by statisticians of the importance of inter-area comparison, initiated a development of significance to demography.[20]

The first basic improvement following Malynes was made in 1693 by the prolific English mathematician, Edmund Halley, in his measurement of the counties of England and Wales. Halley's tables gave county areas to thousands, and, in the case of one small county, even to hundreds of acres. He records the first known use of the "weighing method," and the use of a control check, consisting of measuring the area of England in its totality and then comparing the total to the sum of the county parts. Halley's areas were sufficiently accurate to be used for approximately 100 years, being quoted without mention of source in the *London Magazine* as late as 1772.

The unique character of Halley's weighing method deserves extended quotation from his letter to John Houghton:

The method I took for doing it was, by weighing in nice *scales* that part of the *sheet map of England*, and copied from Mr. Adams' (which I esteem the best) that represents the *land*, and comparing the weight of the whole with that of a circle taken out of the middle of the *map* whose diameter was 138⅔ miles, or two degrees, which was the greatest that the Kingdom can afford; being so much between the Seavern sea in the *West*, and the inlet by *Lyn Regis*, in the East. . . . I found that the *land* of the whole *map*, together with the Isles of *Wight*, *Anglesey* and *Man* weighed just four times as much as the said circle; and consequently that the said kingdom were four times as many as in the circle, which are by computation 9,645,000,[21] whence the whole kingdom must be 38,660,000 acres, and this I believe to be no wise conjecture, but from it you are to deduct from the roads, rivers, and unimproveable mountains according to Judgement. When my hand was in, I thought it might tend to the same end, or be otherwise serviceable to you, to give the acres of each *county of England*, which I have derived from the same method of weighing, having a six sheet *Map* in pieces for that purpose, in which each 40,000 acres weighed about a grain. In this I took care to avoid two inconveniences; the one, that the *map* consisting of several sheets of paper, they were found to be of different thickness or compactness, so as to make a sensible difference, which obliged me to examine the proportion between the weight and acre in each *Sheet:* the other was, that the moisture of the air imbibed by the paper, did very notably increase its weight; which made me very well dry the pieces before I weighed them, so I might be assured there was no error upon that account; and in so doing, I found that in a very few minutes of time, their weight would sensibly increase by their reimbibing the humidity out of the air. This method I conceive exact enough for the uses you design, and that I have not much erred, will appear by the consent of this trial with the former. . . . In all 39,938,500 acres.[22]

The publication in 1729 by Thomas Templeman of areas for all continents of the earth, using square nautical miles, had an important effect on the development of area measurement. He clearly makes geography and geographers responsible for the measurement of geographic area.

The Science of Geography is too Excellent and Entertaining and the Usefulness of it too Universal, to need my Recommendation; or indeed any study'd Motives to induce our British Youths to make the Globes and Maps one of their first Amusements.

Without this Knowledge, their study of Sacred or Prophane History, Travels or Voyages, would be dark and confus'd, their Ideas huddled together and indigested, and their precious and invaluable Time wasted to little or no manner of Purpose.

But notwithstanding this Science is a Qualification so absolutely necessary, it has never hitherto been rightly understood; we have as yet, but a very imperfect Knowledge of the Area, or superficial Content of every Kingdom, Province, and Island, either in square Miles, or Acres, which must first be settled, before the Extent of a Country, or the just Proportion which it bears to any other, can with any Exactness or Accuracy be determin'd.

For these Two thousand Years last past, this Important Article has remain'd a Secret; our Statesmen, Politicians, Travellers, and even Geographers themselves, have wander'd in the Dark, and their sentiments have been meer Conjectures, and vastly distant from the Truth.

The latter indeed (in some Measure to compensate for this Imperfection) have given us the Length, Breadth, and Circumference, or the Situation of Places, between such and such degrees of Latitude and Longitude; but supposing their Admeasurement even carefully and truly taken from the best Maps, yet the irregular Form of all Countries render their Labours very incorrect, I had almost said wholly useless.

But admit that every Kingdom and Province was a true Square or Circle, yet this concession would fall vastly short of answering the End propos'd; nor can the Proportion, which one

[20] Entirely independent of these developments in England, mention should be made of the work of the German geographer E. W. Happel (1687) who computed the area of all countries, totals for the continents, and calculated the land and water proportions of the earth. His work which probably used the polygonic method gave areas in square degrees, and although it might have been an important forerunner of a future development, it actually went unnoticed by all but a few of his contemporaries.

[21] Obviously this amount represents a misprint and should read 9,665,000.

[22] Houghton, John. *Husbandry and Trade Improv'd: Being a Collection of many valuable Materials relating to Corn, Cattle, Coals, Hops, Wool, etc.* The Second Edition (London: Woodman and Lyon, 1728), Number XXIV, 67–70.

A German geographer, H. Scherer (1710), also used the weighing method. A translation of Schmiedeberg's account is interesting: "After he had cut Europe out of a map of thick paper, using a knife along its boundaries, he constructed out of this same map larger and smaller squares which represented 1,000, 100, 20, 10, etc., square degrees. In this way he determined a complete weight scale, and then weighed it, placing Europe on one weight pan and the necessary number of weight squares on the other. He calculated through this weighing that Europe had a size of 1,051 square degrees." Roger Long (1742), had the unique idea, in determining the proportion of land to water surface on the earth, to achieve true area by cutting the land from the water of a paper globe-covering and then weighing the parts. Further the weighing method was used by a number of Europeans working in the area measurement field, and one, G. C. Oeder, in 1777, considered that he originated the method.

Country bears to another, without the area be possibly determin'd.[23]

Templeman's area tables surpassed all previous publications in their detailed coverage of the countries of the earth. He linked the areas of countries to short summaries of their distinguishing and important characteristics. In so doing he gave weight to the importance of inter-area comparison and the man-land ratio suggesting the chore concept of population density.

Templeman does not state what method he used to measure his areas. He does give some idea of how much time was consumed by the work and he states whose maps he used . . . "about eight or nine years ago (for my own private Amusement and Information, without any View at that time, of appearing in Publick), I took the pains to measure all the Kingdoms, Provinces, States and Islands in the World, by Mr. Moll's Two Sheet Maps, and Took the Area in Square Miles . . ." Templeman makes no mention of Malynes, but since his investigations uncovered the continental comparisons of Plinius it is difficult to believe that he was unfamiliar with the work of his near contemporary.

Credit is due Templeman for use of nautical square miles which rendered his tables of international value. The actual method which he used to measure area is in doubt. One might judge from the large scale maps produced by Moll, subdivided into equal area squares, that he probably used some adaptation of the "square-net method." [24]

[23] Templeman, Thomas. *A New Survey of the Globe: Or an Accurate Mensuration of All the Empires, Kingdoms, Countries, States, principal Provinces, Counties and Islands in the World* (London: J. Cole, 1729), p. i. See Appendix A for a facsimile reproduction of a portion of this book.

[24] The "square-net method" consisted of laying out the figure of the country to be measured onto squares, counting the squares wholly within the country, and estimating the partial squares to obtain the total area. Notwithstanding Schmiedeberg's assumption that Templeman used the square-net method, because of the manner in which the Moll maps were laid out into squares, his opinion might be taken as pure conjecture. It might be argued that Templeman used the polygonic method and a nautical mile scale to obtain his areas in nautical square miles, which did not correspond to the squares of 9 and 25 statute square miles shown on the Moll maps.

Templeman's areas were extensively quoted by statisticians and demographers, with or without reference to source. His summary method of presentation was widely appreciated. Shortly after the appearance of Templeman's areas, the advanced English techniques of statistical presentation, exemplified by numerous government publications, were transplanted to the Continent and further stimulated by the work of the German, J. P. Süssmilch. In Süssmilch's analysis of population density developed in his fundamental work, *Die göttliche Ordnung*,[25] his densities are based on Templeman's areas. The work of Süssmilch was widely noted throughout Germany. Through his stimulation, particularly of the prolific author Anton F. Büsching,[26] the measurement of area became a required part of geographic analysis.

Area measurements, with emphasis on Europe, but covering all countries of the earth, were undertaken by geographers far too numerous for separate citation. For nearly a century after the geographical writings of Büsching the methods used for obtaining areas remained laborious and inaccurate. Great need stimulated invention. Simultaneously, though slowly, the development of geodetic area tables for quadrilaterals of latitude and longitude and the invention and perfection of the linear recording planimeter were to solve the problem of measuring the area of map surfaces depicting irregularly shaped geographical regions.

[25] Süssmilch, Johann Peter. *Die göttliche Ordnung in den Veränderungen des menschlichen Geschlects aus der Geburt, Tod, und Fortpflanzung desselben erwiesen*, (Berlin: J. C. Spener, 1741). For a short English summary of Süssmilch's many contributions to statistics and demography, refer to Crum, F. S. "The Statistical Work of Süssmilch," *Quarterly Publications of the American Statistical Association*, (September, 1901). A fruitful geo-historical study might be undertaken to trace the origins and development of the man-land ratio concept of population density, for which study one could turn with profit to the work of J. P. Süssmilch.

[26] Büsching, Anton F. *Neue Erdbeschreibung*, (1754). French and English editions of the mammoth, multivolume work were also printed, and apparently were widely read throughout Europe and the United States.

Chapter V. A Solution for the Problem of Area Measurement

WITHIN one hundred years after the work of Süssmilch and Büsching the problem of how to accurately measure geographical area was essentially solved. The solution did not result from a clear cut sequential development. It was necessary for geodesy and cartography, to provide the map foundation upon which all geographical area measurement rested.

Maps were steadily improved during this one hundred-year period. Sextants, chronometers, and a knowledge of astronomy made possible accurate latitudinal and longitudinal determinations needed to locate physical features on a map. Much remained to be desired, a fact well understood by contemporary scholars. In 1798 it was appropriately remarked by von Zach, "in what uncertainty we still work even at this hour concerning the true size and position of the Black and Caspian Seas? . . . Even in the year 1768 the longitude of such a famous point as the Island of Gibraltar and the not less important city and harbor of Cadiz were still in doubt to within one-half of one degree." [27]

The recognition at the end of the eighteenth century of the necessity for making area measurements on equal area maps was an important forward step of credit to von Zach who worked intensively on this question. He stated, "How many geographers and statisticians take, for example, no consideration of the character of the projection of their maps and calculate their area entirely according to the scale of the map as though it were a flat geometric figure." [28] The first important theoretical treatment of the subject of equal area maps was made by Heinrich Lambert in 1772, and projections of outstanding value were developed by K. B. Mollweide and H. C. Albers in 1805.[29]

The next advance in measuring irregular geographic areas, which received recognition and use during this period, had been made by Thomas Simpson in 1743.[30] The "Simpson Rule," which he and others substantiated, provided a valuable means for computing the areas of irregular boundary portions of geographical regions.[31] The Simpson Rule is applied as follows: The area of the major portion of a region is calculated by means of the polygonic method, to a straight base line along an irregular boundary. Then by projecting equally spaced perpendicular lines from this base line to the irregular boundary, obtaining a properly weighted average length of these perpendiculars, and multiplying this average by the length of the base line used, an area is obtained which is proportional in accuracy to the spacing of the perpendiculars (fig. 2).

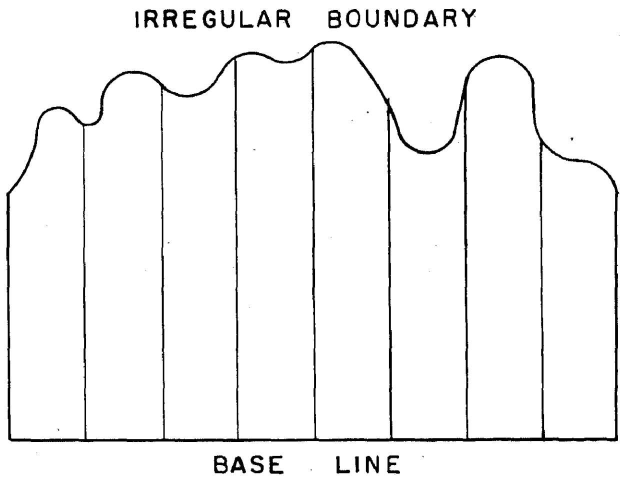


FIGURE 2.

The next development was the so-called "net planimeter." This device had an obscure origin, but evidently belongs to the eighteenth century. It consisted of a wooden frame, subdivided into 100 squares by threads or hairs spaced according to the map scale, or a glass plate marked with 100 squares and supplemented by a smaller glass plate subdivided into small squares each one-hundredth part of the larger squares. A net planimeter was placed over the area to be measured and the squares and their subdivisions were then counted and the irregular portions estimated. This greatly increased the speed of measurement, although

[27] *Allgemeine Geographische Ephemeriden,* (1798), Band I, Einleitung, 21.

[28] Von Zach, *Monatliche Correspondenz zur Beförderung der Erd und Himmelskunde,* (1800), 169.

[29] Gelcich, E. *op. cit.* This author gives an account of the development of equal area projections and their relation to area measurement.

[30] Simpson, Thomas. *Mathematical Dissertations on Physical and Analytical Subjects.* (London, 1743).

[31] It should be noted that during this period the weighing method was tested and retested and found of rather limited value except as a supplementary means for obtaining the area of irregular boundary segments. For this use it was recommended by von Zach and others. The polygonic method too was pursued with diligence and with customary complaints concerning its laborious character and inaccuracies.

the results still were not in terms of a direct reading. The main contribution made by the net planimeter was not the measurements which it helped to accomplish, but the easy adaptation of its principle by geographers in the use of geodetic area tables for quadrilaterals of latitude and longitude as a means for determining the area of geographic regions.

The first direct contributor to the vital development of geodetic area tables was H. E. Kästner. In 1755 he roughly assumed Germany to be bounded by the 46th and 55th parallels and the 24th and 37th meridians. On this basis he calculated its area as though it were a spherical rectangle.[32] Following the work of Kästner it was a natural next step for Professor G. S. Klügel, in 1781, to have his student Zitting prepare a geodetic area table for 30-minute quadrilaterals.[33] Klügel deserves credit for advising those who might use this table to calculate areas from 30-minute parallel strips traversing the entire length of the country and to geometrically calculate the boundary remnants. A table for five degree quadrilaterals was contributed by J. E. Bode in 1783.[34] Later Ebling and von Zach make a clear statement of the advantages inherent in the use of geodetic area tables and they contributed a refinement for the method by preparing quadrilaterals of 15 seconds in size on transparent paper so that these might be used in a manner similar to a net planimeter; thus making it possible to estimate area by inspection to $\frac{1}{16}$ of a square minute.[35]

European scholars soon recognized the necessity of calculating geodetic area tables which gave proper consideration to the oblateness of the earth. The necessity for this consideration was pointed out by Klügel in 1787.[36] He suggested how the work might be done, but several mathematical uncertainties, and the labor involved, probably prevented the effort. Von Zach too, in 1880, complained that no work had been undertaken to prepare such tables. While this state of affairs existed in Germany, two Russian mathematicians in 1787 and in 1795 prepared geodetic area tables which considered the oblateness of the earth. The first of these, W. L. Krafft, using an integrations method and the fraction 1/200 for ellipticity, prepared a table for 30-minute quadrilaterals and used it to measure the area of the Russian Empire.[37] He was followed by Storch who used Newton's value of 1/230 for ellipticity and remeasured the Russian Empire.[38]

Painstaking measurements by French, English, and other European cartographers, as already noted, followed in an effort to establish the precise variation in the length of degrees at various latitudes. Finally, the geodetic calculations of F. W. Bessel (1841) and A. R. Clarke (1866), based on these measurements, provided essentially correct earth dimensions and laid the foundation for accurate geodetic area tables. These tables were not long in preparation. In 1845, Wohlstedt prepared tables based on the less accurate Bessel dimensions of 1837. These were recalculated into Russian units by F. G. W. Struve and G. Schweizer,[39] who used them to determine the area of Russia. In 1858, A. Steinhauser of Vienna used the Bessel dimensions of 1841 and prepared usable tables in square German miles for quadrilaterals of one degree of longitude and 30 minutes of latitude.[40]

Accurate geodetic area tables of themselves did not provide a solution for the problem of measuring the areas of irregularly shaped geographical regions as recorded on maps. Such regions, by not fitting quadrilaterals of latitude and longitude, required some mechanical measurement device for making direct area readings. Fortunately this device, the linear recording planimeter, had a development concurrent to the preparation of accurate geodetic area tables.

The German mathematician, J. M. Hermann (1814),[41]

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[32] Lulofs, J. *Einleitung zu der mathematischen und physikalischen Kenntniss der Erdkugal.* Aus dem Holländischen übersetzt von A. G. Kästner, (Göttingen u. Leipzig, 1755), II, 156, Anm. Kästner elaborated on the work of the Dutchman with his own observations from which the above is taken from Schmiedeberg. The works of Clauvius (1612), Tacquet (1707), and Liebknecht (1707) are also related, in a minor way, to this development.

[33] Klügel G. S. "Tafel des Inhaltes aller Zonen einer Kugel von halben zu halben Graden der Breite," *Astronomischen Jahrbuch für das Jahr 1784,* (Berlin, 1781), 174.

[34] Bode, J. E. *Beschreibung und Gebrauch einer neuen Weltkarte,* (Berlin und Stettin, 1783), 59 and Tabelle 14.

[35] von Zach, *op. cit.,* and other editions. J. T. Mayer also contributed to a mathematical explanation of the validity of using geodetic areas for quadrilaterals of latitude and longitude. See Mayer, J. T. *Gründlicher und ausführlicher Unterricht zur praktischen Geometrie,* IV, (Erlangen, 1794), 188–207. Th. Willers makes several minor additions to the statements made by W. Schmiedeberg in discussing these developments. Willers, Th. "Zur Geschichte der Geographischen Flächenmessung seit einführung des Planimeters," *Petermanns Mitteilungen,* Ergänzungsheft, Nr. 70 (191), 24. This 96 page discussion provides the primary basis for the remainder of this Chapter and Chapter VI. References used, when available, were checked, and in certain instances supplemented by additional material.

[36] Klügel, G. S. "Berechnung der Zone dem Äquator und einem Parallel kreise," *Astronomisches Jahrbuch fur das Jahr 1790,* (Berlin, 1787), 243.

[37] Krafft, W. L. *Nova Acta Academie Scientiarum Imperialis Petropolitanae,* Tom I, (Petersburg, 1787), 389.

[38] Storch, *Statistiche Ubersicht der Statthalterschaften des Russischen Reiches,* (Riga, 1795).

[39] *Bull. de le Classe phys.-math. de l'Acad. des Sciences de St. Petersbourg,* Bd. IV, (1845), 362. In 1837 Bessel published the fraction 1/300.7 for the ellipticity of the earth, which he corrected in 1841 with the fraction 1/299.15.

[40] *Petermanns Mittelungen,* (1858), 465–68. Th. Willers provides additional detail concerning other famous geodetic area tables, namely: Hermann Wagner's tables of 1870, based on Bessel (1841), and using square meters; I. A. Strielbitsky's tables of uncertain date, based on Clarke (1866), and used in his measurement of the Russian Empire, released in 1874; the F. De Y. Carpenter and Henry Gannett formula for tables based on Clarke (1866), and used in the measurement of the United States as published by the Census Bureau in 1881; and the R. S. Woodward tables of 1894, based on Clarke (1866) and released as the Smithsonian Geographical Tables.

[41] Bauernfeind, C. M. "Zur Geschichte der Planimeter," *Dinglers Polytechn. Journ.* CXXXVII, Heft 2.

is credited with discovering the principle underlying the linear recording planimeter. The work of Hermann was followed by an independent invention by an Italian T. de Gonnella (1825),[42] of a linear recording planimeter based on the same principle. Unfortunately the operation of this planimeter, constructed for Gonnella by a Swiss engineer, J. Oppikofer, proved complex. This fact in combination with the high cost of the instrument, confined its use to a few working models. The design and principle of the Gonnella planimeter became known to the German, Jacob Amsler, in 1849. In 1854, Amsler released his improved design for an instrument which became known as the "polar planimeter".[43]

The operation of the polar planimeter is based on the fact that the area swept over by a moving line is a function of the length of the line and the perpendicular distance traveled by its midpoint. That this is sometimes true is made apparent by two simple examples: Let AB move into the position A′B′ (fig. 3). At all

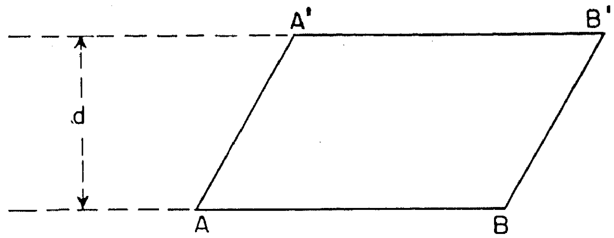


FIGURE 3.

times the line is parallel to its starting position; the point A moves along the straight line AA′. The area swept over by the moving line is the parallelogram AA′B′B, the area of which is given by plain geometry as the product of the length of one side and the perpendicular distance between it and the opposite side. The distance the midpoint has traveled perpendicular to the line, designated as d, is equal to the perpendicular distance between AB and A′B′, and the area of AA′B′B = d × AB.

Let AB move into the position AB′ so that the point A does not move, and the point B travels along the arc of a circle with its center A and its radius AB (fig. 4). The distance, d, which the center of the line P, travels perpendicular to the line is the length of the arc of

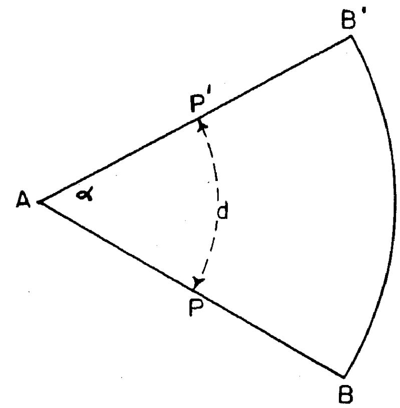


FIGURE 4.

the smaller circle PP′. The area swept over by the line is the sector of the circle ABB′ whose area is given by plain geometry as:

$$\frac{\alpha}{360^\circ}\,\pi(AB)^2$$

or

$$\frac{\alpha}{2}\,(AB)^2$$

when α is measured in radians. Since P is the midpoint of AB, $AP=PB$ and $2AP=AB$. Then,

$$ABB'=\frac{\alpha}{2}\times 2AP\times AB=\alpha AP\times AB$$

But

$$\alpha\times AP=d,$$

and

$$ABB'=d\times AB.$$

Integral calculus is necessary to prove the general statement that the area swept over by a moving line is equal to the product of the length of the line and the perpendicular distance traveled by the midpoint. The line AB, midpoint P, sweeps over the area ABB‴A‴ (fig. 5). Any very small portion of this area may be treated in the following way. Consider the area A′B′B″A″.

$$A'B'B''A''=CB'B''-CA'A''$$

As in the example given above:

$$A'B'B''A''=(CB'^2-CA'^2)\frac{\alpha}{2}$$

$$=(CB'-CA')\,(CB'+CA')\frac{\alpha}{2}$$

But $CB'-CA'=A'B'$, and $CB'+CA'=2CP'$. Therefore:

$$A'B'B''A''=A'B'\times 2\times CP'\times\frac{\alpha}{2}=A'B'\times CP'\times\alpha.$$

[42] Favaro, Antonio: *Beitrage zur Geschichte der Planimeter*, (Wein: R. von Waldheim, 1873). No description has been given here of the "harp planimeter" invented by Oldendorp in 1825 or the "ring planimeter" invented by Westfeld in 1826. These are described by Th. Willers and W. Schmiedeberg. The former planimeter because of its laborious operation, and the latter because of the inaccuracies resulting from its operation, though of passing interest, were of little consequence in the development of the linear recording planimeter; a planimeter so named because it records the area of a figure when a pointer is traced around its perimeter. (Called Umfahrungsplanimeter in German, a good name for which, unfortunately, there is no adequate English translation.)

[43] The name, polar planimeter, results from Amsler's introduction of a polar arm attached to a wheel carriage, the far end of which arm is pinned or fastened to the map surface and acts as a pole or center for the swing of the instrument. Amsler, Albert: "Das Planimeter und seine Erfindung," *Zeitschrift des Vereins Schweizer . Konkordatsgeometer*, V, (1907).

But as in the example above,

$$CP' \times \alpha = \Delta s; \text{ and } A'B'B''A'' = A'B' \times \Delta s.$$

Summing all such areas:

$$ABB'''A''' \int = AB \times ds = AB \times s.$$

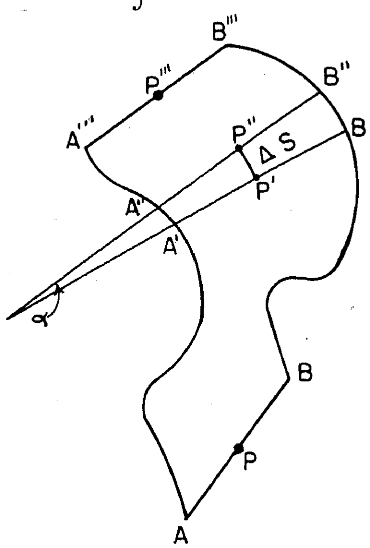


FIGURE 5.

Two rods enter into the construction of the polar planimeter, the first rod, represented by CH has a pin at C which holds this point fixed during any one measuring operation (fig. 6). The other end is hinged to the

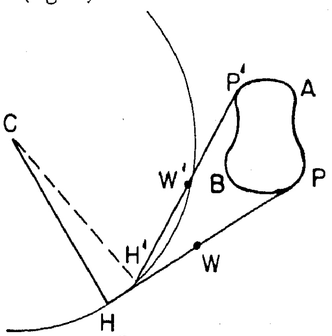


FIGURE 6.

second rod at H. It is apparent that the point H will at all times lie on the arc of a circle with its center at C and radius CH. The other end of the second rod, P, is the tracer point which is moved around the perimeter of the area to be measured until it is returned to its starting position. A wheel, W, is at the center of the perpendicular to HP. As HP, which is regarded as a moving line, is moved across the paper, this wheel turns the amount the point W travels perpendicular to HP and slips the rest of the distance. The distance traveled by W, perpendicular to HP, is therefore $2\pi rn$ where r is the radius of the wheel and n is the number of revolutions. The area swept over by HP is seen to be $HP \times 2\pi rn$. As P is moved through A to P′, the area swept over by HP is PAP′H′H; as P′ is moved through B and back to P the area swept over is HPBP′H′, but since the wheel will be turning in the opposite direction, this area is subtracted and substitution in the above formula will yield the area PAP′B. This is true no matter what point on the perimeter is chosen as the starting and terminating point. It is also apparent that tracing the perimeter in one direction will yield a positive result and reversing the direction will yield an equal but negative answer. (Fig. 7 shows a photograph of a standard polar planimeter).[44]

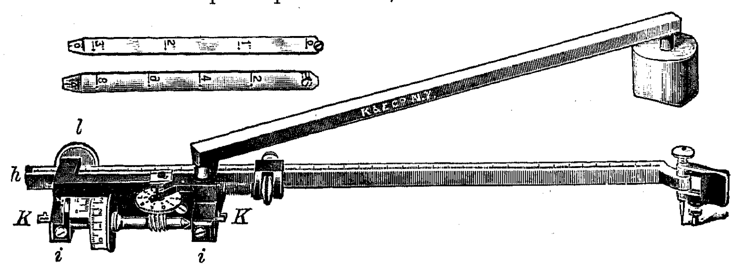


FIGURE 7.

The simplicity of the polar planimeter, its low construction cost and the accuracy of the results obtained from its use,[45] effectively solved, in combination with the use of geodetic area tables for quadrilateral of latitude and longitude, the problem of accurately measuring irregular geographic regions portrayed on maps.[46]

[44] For additional explanations for the operation of the planimeter, refer to Hammers, E.: "Planimeter" *Otto Lüger: Lexikon der Gesamten Technik,* Bd VI, (Stuttgart: W. Leipzig, 1898), 775; Tennant, W. J.: "The Planimeter Explained Simply Without Mathematics," *Engineer,* Vol. 95 (1903), 75; and Granville, W. A., Smith, P. F., and Longley, W. R.: *Elements of Differential and Integral Calculus,* (Boston: Ginn and Company, 1934), 492 and 493.

[45] Under controlled conditions results are accurate to even less than .15 of one percent. Errors of .15 to 1.0 percent result in ordinary hand operations, depending on the size of the area to be measured, the character of the map paper, the skill of the operator, and a number of other possible variables. For a rather full discussion of this question refer to Th. Willers.

[46] Although the polar planimeter of Jacob Amsler essentially solved the problem of area measurement, progress continued to be made in the construction of various types of planimeters. These improved instruments (the "roll planimeter," the Cordai planimeter, the "compensating planimeter," and the Hohmann planimeter), increased the measurable area, made it possible to set planimeters to the scale of maps to provide for direct area readings, and in general to increase the accuracy and utility of the instrument. These developments are discussed by Th. Willers and abundant references are made to original sources.

Because of its practical value to the geographer, attention should be directed to the so-called "hatchet planimeter," discovered in 1889 by Pryzt, a Dane. This planimeter, readily constructed from 3/16 inch drill rod stock at the cost of a few cents, is discussed by Th. Willers; explained mathematically by Satterly, J.: *The Hatchet Planimeter,* (Toronto: University of Toronto Library, 1921); and its construction and use further explained in a circular of the *Soil Conservation Service of the Department of Agriculture,* Dickerson, L. M., "The Hatchet Planimeter," Regional Circular 196 (July 2, 1940), Region 3, Dayton, Ohio.

Other references of interest on the development of devices for measuring areas are: Schlink, F. J.: *Area Measurement of Leather,* U. S. Bureau of Standards, Technologic Papers (April 24, 1920), 1–47, discussing the "weighed-pin" and "parallel-roller" machines; and Gradstein, S.: "Simple Photo Planimeter," *Review of Scientific Instruments,* Vol. 5, (September, 1934), 311, describing a photoelectric cell device for scanning and recording the area of leaves. It is humorous to note that a Government employee recently "invented" the weighing method for determining area.

Chapter VI. Modern European Areas

THE modern period of European area measurement begins with Friedrich Bernhard Engelhardt and the publication of his epochal 123-page volume devoted entirely to area statistics for the individual countries of Europe and for the remainder of the earth.[47] Credit for this contribution likewise belongs to the farsighted statistician, Karl F. W. Dieterici, Director of the Kgl. Preuss. Statistichen Bureaus zu Berlin, for establishing a cartographic division in his bureau several years earlier and giving Engelhardt his area measurement assignment.

Engelhardt's work was based on the first systematic selection of the best maps then available covering all countries of the earth. His work gave greatly expanded detail, and it used geodetic area tables which considered the oblateness of the earth.[48] However, the linear planimeter was not yet in common use and it was necessary for Engelhardt to employ the polygonic method to compute the irregular fringe areas of all countries.

Engelhardt's place in the history of the measurement of geographic area is secure. Other workers subsequently carried out individual measurements with considerably more accuracy, using improved geodetic area tables and linear planimeters, but, in the detail of worldwide coverage all reduced to a single measure (German square miles), Engelhardt's work has not since been equalled.

The next developments were made by staff members of the Geographischen Anstalt von Justus Perthes in Gotha, Germany. This establishment developed from an eighteenth century book publishing house which early specialized in geographical area statistics. It released the bulk of these statistics, with careful reference to source, in an almanac called the *Gothaische Hofkalander*.[49] In 1816 this almanac was taken over by the publishing house of Justus Perthes of Gotha where it was consistently improved as a reference book based on secondary source material.

The Geographischen Anstalt von Justus Perthes in Gotha underwent a number of fundamental changes during the middle of the nineteenth century. It moved to its own building, and added professional and technical personnel to its staff. It developed a library and map collection. In 1854 it was joined by August Petermann who founded his now world-famous, *Geographischen Mitteilungen*. In 1856 Ernst Behm joined the staff of the Anstalt and began his bibliographical research culminating with the publication of the first *Geographische Jahrbuch* in 1866. Important advances in the development of modern area measurement were indicated for this geographical setting.

Probably the first important development at the Anstalt was the painstaking work of Behm who combed the literature for the statistics of all countries and published them with reference to source and the apparent reliability of each source. Engelhardt's areas were corrected as more reliable information became available; there being as many as 300 corrections based on planimeter measurements indicated by Behm in his first release of 1866.[50] In addition to Behm's invaluable checking of sources to establish their reliability, the staff of the Anstalt made contributions in the form of actual area measurement. Ernst Debes and F. Hanemann in 1865 released their measurements for the area of Australia, based on a nine sheet map on a scale of 1: 3,500,000. Their work records the first combined large scale use of geodetic area tables (for 30-minute quadrilaterals) and the linear planimeter to measure coastal fringes. In 1869 F. Hanemann used the planimeter to measure Alaska with its coastal islands.

[47] Engelhardt, F. B. *Der Flächenraum der einzelnen Staaten Europas und der übrigen Länder der Erde* (Berlin, 1853). The foreword to this work, written by Dieterici gives such a fine summary of the then current importance of area measurement to the statistical profession that it deserves quotation.

"Es ist bei statistischen Betrachtungen von grösster Wichtigkeit, die Raumverhältnisse eines gegebenen Landes genau zu übersehen. Es genügt nicht, das einzelne Land, auf welches sich eine besondere Betrachtung bezieht, nach seiner Grösse genau zu kennen, es ist nötig, auch andere Länder zu vergleichen, um sich ein klares Bild obwaltender Verhältnisse entwerfen zu können. Bei solchen Arbeiten und Vergleichungen fühlte ich seit langer Zeit das Bedürfnis, einen möglichst sicheren Anhalt zu haben über die Grössenverhaltnisse der verschiedenen Staaten Europas, aber auch der übrigen Länder auf der Erde. . . . Es finden sich nun zwar die Grössenverhaltnisse der verschiedenen Staaten in geographischen Werken angegeben; der Weimarsche Genealogisch-historisch-statistische Almanach für das Jahr 1848 stellt sie recht übersichtlich zusammen, aber teils ist die Darstellung, wenigstens in neuerer Zeit, nicht vollständig, teils schien es mir wichtig, dass eine genaue Revision der vorhandenen Angaben, da sie mehrfach wohl ohne nahere Kritik aus einem Werk in das andere übernommen wurden, angestellt werde."

[48] His tables used the fraction 1/310 for the ellipticity of the earth.

[49] The Geographischen Instituts zu Weimar, which published the Weimarische Geneologisch-historisch-statistische Almanack had a parallel development and made an important contribution as mentioned in the quotation from Dieterici (see footnote 47), but shortly after 1848 the publication of this almanac was discontinued.

[50] Behm's research for the *Geographische Jahrbuch* encompassed the entire field of geography and statistically was equally concerned with the population of countries.

In 1868 the Anstalt was fortunate to secure the services of Hermann Wagner, who took over the direction of the *Statistischen Jahrbuchs* in the *Gothaer Hofkalender*. For his contribution he recomputed all areas published in the yearbook into square kilometers, giving the reduction factors used and full reference to source. In 1869 he began an exhaustive investigation of the reliability of the official areas given for the subdivisions of Germany,[51] probing sources, correcting errors and making remeasurements. The results of this work were presented to the members of the Internationalen Statistischen Instituts, meeting in Berlin in 1903.[52]

Hermann Wagner took an active part in bringing about the adoption of the metric system in Germany. In addition to his innovation of publishing areas in square kilometers, he contributed a recalculation in 1870 of Bessel's earth dimensions according to the metric system.[53] These metric dimensions were used in the preparation of a number of geodetic area tables. These tables led to a great many significant European remeasurements, some of which, for the subdivisions of Germany, were made by Wagner.

Wagner's final contribution to area measurement was the periodic publication of book sized supplements to *Petermanns Mitteilungen*, entitled *"Die Bevölkerung der Erde."* These supplements brought order out of the confusion resulting from increasingly numerous, yet conflicting areas for the countries of the world. Fourteen supplements have appeared since the first initiated by Wagner with the collaboration of E. Behm in 1872.[54] The purpose of this periodic publication, as stated in the first issue, was "an annual review of new area measurements, boundary changes, censuses and population estimates for the entire earth." It was Wagner's long range objective to obtain the areas of all countries and continents and a total for the earth by following uniform principles involving geodetic area tables based on one set of earth dimensions and planimeter measurements applied to reliable maps. In an important measure, within the limits imposed by the availability of reliable maps, Wagner's objective has been fulfilled.

Mention should be made of the specific work done as a direct result of the stimulation of Hermann Wagner. In 1878 O. Krümmel remeasured the oceans of the earth and found them to be approximately 8 million square kilometers smaller than the area then assumed to be correct.[55] His measurements led to an exhaustive reevaluation by Wagner and Behm of the areas of all continents and the entire surface area of the earth. The work of Krümmel was followed by that of B. Trognitz who released a remeasurement of Africa based on Stanfords Library Map on a scale of 1: 5,977,382 in 1891,[56] and tested his results by a remeasurement of Africa in 1893 based on the 1: 4,000,000 Habernicht map.[57] In addition Trognitz is to be credited with at least 250 painstakingly executed measurements which saw publication in 1891.[58] Finally a number of reliable areas resulted from the measurements of H. Haack and A. Supan.[59]

An outstanding contribution to the development of modern European area measurement was made by a Russian army officer, Ivan A. Strielbitsky with the publication in 1882 of his detailed volume *La Superficie de l'Europe.*[60] This work resulted from the 1869 meeting of the *International Statistical Congress*, where the area measurement assignment was given Russia, as part

[51] Wagner, H. "Die Deutsche zollverein und die Freihafengebiete Deutschlands nebst bemerkungen über die Arealangaben der deutschen Staaten" *Petermanns Mitteilungen* (1869), 247–55.

[52] *Bull. Inst. intern. de Statist.* XII and the *Geographische Zeitschrift* IX (1903), 685–93.

[53] Wagner, H. "Dimensionen der Erde nach Bessels Elementen im Metischen Masse, vorzüglich für die Bedürfnisse der Kartographer," *Geographischer Jahrbuch* III, 1870.

[54] Seven of these supplements (Ergänzungshefte), numbers 33, 35, 41, 49, 55, 62, and 69 appeared under the authorship of Behm and Wagner between 1871 and 1882; two, numbers 101 and 107 under the authorship of Wagner and Supan between 1891 and 1893; three, numbers 130, 135, and 163 authored by Supan between 1899 and 1909; and a recent continuation, number 212, was authored by Tamss in 1932. Supan gives credit to Wagner as the guiding spirit for this work in the introduction of the 10th edition of 1909. "Es unterliegt für mich keinem Zweifel das hauptsächlick Wagner es war der diesem Sammelwerk den Giest des strengen Kritizismus eingeflanzt hat." Then too, the year span of Wagner's authorship exceeds that of Behm who died in 1884.

[55] Behm, E., and Wagner, H. "Die Bevölkerung der erde," VI, *Petermanns Mitteilungen*, Ergänzungsheft Nr. 62 (1880), 3.

[56] Behm, E. and Wagner, H. "Die Bevölkerung der Erde," II, *Petermanns Mitteilungen*, Ergänzungsheft 35 (1873), 96.

[57] *Petermanns Mitteilungen* (1893), 120.

[58] Wagner, H. and Supan, A.: "Die Bevölkerungder Erde," VIII, *Petermanns Mitteilungen*, Ergänzungsheft Nr. 62 (1891).

[59] *Petermanns Mitteilungen* (1897), 96, 184, 236; (1898), 107, 185; and (1900), 167 and 192.

[60] *Superficie de l'Europe Etablie par J. Strielbitsky*, Publications du Comito Central Russe de Statistique (St.-Petersbourg: Trenke et Fusnot, 1882) 228 pages. According to the Library of Congress reference to this work the author's name is Ivan Afanzsevich Strielbitsky; whereas T. Willers refers to him as Johann A. Strelbitsky. Area measurement in Russia has had a long and venerable development culminating in Strilbitsky's publication of 1882, which has not since been equalled in scope. Th. Willers gives a short discussion of this development. See: Struve, F. G. W. Über die Flacheninhalt der 37 westlichen Gouvernements und Provinzen des europäischen Russlands," *Bull. Acad. Sci. St. Petersbourg*, Cl. phys.—Math., (1845) 429–46, telling of the area measurements of G. Schweizer; Schweiber, G. "Arealbestimmung des Kaiserreichs Russland mit ausnahme von Finnland und Polen," *Bull. Acad. Sci. St. Petersbourg, Classe des sciences hist. philol. et polit.*, XVI (1859), Numbers 24–27, pages 369–431 and Numbers 30–36, pages 465–561, recording one of the first uses of Amsler's polar planimeter; and Strielbitsky, I. A. "Berechunung der Oberflache des Russichen Reiches nach Einteilung und Bestand unter der Regierung zar Alexanders II, (St. Petersburg: Der Druckerei der Kais. Akad. d. Wiss, 1874), viii and 249 pages. The tireless Strielbitsky did not rest with his two contributions but undertook a revision of his 1874 areas. Strielbitsky, I. A. "Berechnung der Oberfläche des Russischen Reiches nach Einteilung und Bestand unter der Regierung des Zars Alexander III. Nebst den russisch-asiatishcen Grenzländer," (St. Petersburg; Der Druckerei des Generalstabs, 1889) xiv and 135 pages.

of a world-wide program to promote statistics. P. V. Semonoff, Director of the Central Statistical Committee of Russia fostered the work undertaken by Strielbitsky, and wrote the preface to his publication.

Strielbitsky's publication of 1882 is divided into an introduction, six sections and an appendix. The introduction describes the methods and cartographic material used, makes observations relative to the areas given, compares the new areas with some previously published, and makes various general observations relative to calculating the areas of geographic regions. As for the six sections, they give in succession, with introductory explanations, the areas of Europe by: countries and administrative divisions; zones of one degree of latitude; river basins, islands, and lakes; and large seas, peninsulas, and gulfs. The appendix gives geodetic area tables by 10 minute and degree quadrilaterals of latitude and longitude, based on the A. R. Clarke's (1866) fraction of 1/294.98 for ellipticity; and a table of area comparisons by square degrees for Clarke, Bessel (1/299.15) and Walbeck (1/302.78). All area statistics are conveniently given in square kilometers, miles, and versts.

Strielbitsky's measurements were based on the best available large scale topographic maps for the countries of Europe. These maps ranged in scale from 1: 40,000 for Belgium and Denmark to 1:1,500,000 for Spain. Substantial areas were not covered by large-scale maps and required filling in with less desirable small-scale maps. As a check, Strielbitsky measured the entire area of Europe on maps drawn to the scale of 1: 420,000. These maps were partly in manuscript form, but printed for Russia, Rumania, Serbia, and Turkey.

Strielbitsky is open to criticism because he did not give a detailed account of each map measured, and therefore it can not be fully established what boundaries were used for the areas of some of the countries. Yet, his publication proved a marked stimulation to area measurement work throughout Europe. So many official or quasi official areas were shown to be in error by the Strielbitsky areas, that each country was promptly stimulated to undertake official remeasurements. Naturally no country would accept areas released by a foreign source. Thus Strielbitsky had an enormous effect in the development of modern area measurement in Europe, even though he lacked the satisfaction of having his areas accepted outside of Russia.[61]

Partly through the stimulation of I. A. Strielbitsky and H. Wagner, work was undertaken by E. Levasseur, L. Bodio, and F. von Juraschek to collect the most authentic, officially released, areas and population totals available for all countries of the world. Their publications appeared in 1886, 1887 and 1902 under the title "Statistique de la Superficie et de la population des Countries de la Terre." [62]

In 1903 Levasseur and Juraschek joined with Wagner in a resolution presented to the Berlin meeting of the International Statistical Institute which though still valid today, unfortunately has not been carried out.[63]

Das Internationale Statistische Institut spricht den Wunsch aus:

1. Dass in allen offiziellen Publikationen, welche das Areal des Staatsgebiets betreffen, spezielle Angaben gemacht werden über die Frage, welche Grenzgewässer in den Ziffern des Areals enthalten sind oder nicht, sei es längs der inneren Landesgrenzen, sei es längs der Meeresküsten.

2. Diese Angaben sollten nicht nur hinsichtlich der einzelnen Verwaltungsbezirke, welche in Frage kommen, spezifiziert werden, sondern auch in betreff der verschiedenen Seebecken and Uferstrecken.

3. Im Falle nur unvolständige Angaben über diesen Punkt vorliegen, werden die Bureaus ersucht, die Ausfüllung der Lücken durch die geeigneten Behörden oder Institute in die Wege zu leiten bzw. selbst planimetrische Ausmessungen, wenn auch von provisorischem Charakter, zu veranlassen.

The work performed by Levasseur, Bodio and Juraschek widened interest in the problems of areas measurement, particularly in France and Italy. Their compilations however did not make any original contribution in the form of more accurate area measurements for any part of the world.

Another major contribution was the remeasurement of the Austro-Hungarian Monarchy brought to completion in 1889 by Franz Stadl and a staff of assistants under the general supervision of Albrecht Penck.[64] This remeasurement was based on a 400 sheet map of the Monarchy on a scale of 1:75,000. The method employed planimeter measurements in duplicate, requiring agreement to within 0.2 of a square kilometer, and final adjustments to geodetic area tables for zones of latitude and longitude. These Stadl-Penck areas were not broken down by the numerous administrative subdivisions of the Monarchy. They were only given for the twenty Krönlander, and Hungary, Fiume and Croatia. Probably for this reason they did not obtain

[61] This applied only to Strielbitsky's areas for political and administrative subdivisions. His areas for seas, lakes, islands, and river basins were accepted for a number of years.

[62] *Bull. Inst. intern. de Statist.* I (1886) releases 3 and 4, (Rome 1887), 1-133 (Introduction et l'Europe); II, release 2 (1887), 163-242 (Afrique, Asie, Oceanie et Amerique; Generalites et resume des cinq parties du monde); XII, release 2 (Rome 1902), i-x and 1-110.

[63] *Ibid*, XIV, release 1 (Berlin, 1905), 42-63 and *Geographische Zeitschrift* IX (1903), 685-93.

[64] "Der Flächeninhalt der Österreichisch-Ungarischen Monarchie. *Sitzb. d. k. Akad. d. Wiss. Wein*, math.—nat. Kl. XCVIII, abt. II, (July 1889).

official sanction and use. However, their value was at once recognized by European geographers, and each remeasurement of the "official areas" more nearly approached the totals recorded by Stadl and Penck.

Modern European area measurement was given further impetus by the work of the Italian Instituto Geografico Militare, completed in 1884.[65] The Instituto was stimulated into action by the writings of L. Bodio in 1873,[66] H. Wagner in 1878,[67] and particularly Giovanni Marinelli in 1883.[68] They pointed out many gross errors in the official Italian area statistics. Marinelli drew his comparisons between the official areas and those published by Strielbitsky in 1882.

The official remeasurement of the Italian Kingdom of 1884 was completed under the direction of L. de Stefani, Chief of the Geodetic Division of the Instituto. He was assisted by two engineers, three calculators, two draftsmen, and two other technicians. The work was done with great care, being based on the best maps available. These maps ranged in scale from 1: 25,000 to 1: 86,400. Each area was measured in original and in independent duplicate (allowing variations up to 1.2 square kilometers). Amsler polar planimeters and geodetic area tables were used, the latter based on the Bessel's (1841) dimensions of the earth. Of particular significance to area measurement is the full discussion accorded the methods employed.[69]

The Stefani remeasurements of Italy, though of a high order of accuracy, were not satisfactory from a statistical standpoint. These remeasurements only gave area totals for the peninsula and the largest islands and did not provide areas for the provinces, circondari, or even the largest administrative subdivision, the compartimenti territorial. Strielbitsky's work of 1882 gave the areas of Italian provinces, but these were considered unsatisfactory because of boundary uncertainties and because his total area for the Italian peninsula was substantially larger than that obtained by Stefani. H. Wagner made a provisional measurement of the Italian compartimenti in 1891, using a small scale map (1: 1,500,000).[70] Later the Statistical Office in Rome used a large scale map (1: 500,000) and planimeter measurements adjusted to Stefani's total areas, to compute the areas of the provinces and their circondari subdivisions.[71]

A final contribution to modern European areas was the measurement of the Republic of France in 1894 by the *Service géographique de l'armée.* It had been well known for years that the official area statistics for France and its administrative subdivisions were grossly inaccurate. As in the case of Italy, the areas of Strielbitsky (1882) served to focus attention on shortcomings of the official French areas. The Italian (Stefani) remeasurements of 1884 likewise gave strong impetus to a movement for improving the situation. This movement was headed by the aforementioned Emile Levasseur.

Through a directive from the Ministry of Commerce and Industry, dated February 19, 1885, a *"Conseil supérieur de Statistique"* was established which undertook as its first assignment "L'étude des moyens á employer pour faire disparaître les divergences dans les evaluations de la superficie territoriale de la France." The Conseil established a subcommittee under the chairmanship of Levasseur to pursue the problem. This subcommittee investigated the numerous inconsistencies in the official areas for the subdivisions of France as released by various governmental agencies. It sought to determine the best maps available for the purpose of remeasuring the areas of France. It went into the question of the ocean boundary of the country. It decided that all departments and arrondissements should be measured and not just totals for the country as had been done in Italy. The final recommendations of the subcommittee were approved by the Conseil, and transmitted to the Minister of Commerce who gave the assignment of remeasuring the areas of France to the Minister of War. The Minister of War assigned the work to the Service géographique de l'armée.[72]

The work of remeasuring the areas of France was started under the supervision of General Perrier, Chef de Service géographique de l'armée. who made a preliminary report to the Conseil in 1887.[73] The work was completed by his successor General Derrécagaix who reported briefly in 1894.[74] These two reports tell that experimental preliminary measurements were followed by final definitive measurements carried out with painstaking care. Geodetic area tables for 10-minute quad-

[65] *Superficie del Regno d'Italia Volutata nel 1884,* (Florenz: Instituto Geografico Militare, 1885) 110 pages with an index map showing the individual maps used in the measurements.

[66] Bodio, L. *L'Italia economica nel 1873,* (Rome, 1874).

[67] Behm, E., Wagner, H. "Die Bevölkerung der Erde V, *Petermanns Mitteilungen* (1878), Erganzungsheft Nr. 55, 19.

[68] Marinelli, G. "Notizie intorno alla questione della superficie d'Italia," *Atti del R. Instituto Veneto,* III (1883), Ser. 6a and "L'Area del Regno d'Italia" *Bolletino della Societa Geografica Italiana* (April 1883).

[69] Various parts of Italy were remeasured as improved large scale maps became available. These new areas were released as appendices to this work in 1896 and 1901.

[70] Wagner, H. and Supan, A. "Die Bevölkerung der Erde" VIII, *Petermanns Mitteilungen* (1891), Ergänzungsheft Nr. 101, 30.

[71] Supan, A. "Die Bevölkerung der Erde," XI, *Petermanns Mitteilungen* (1901) Ergänzungsheft Nr. 135, 50.

[72] *Bull. Conseil superieur de statistique,* No. 1, (Session 1885/86), 184–90.

[73] *Ibid.,* No. 3 (Paris 1889), 63–67.

[74] *Comptes rendus hebd. des séances de l'Acad. des sciences de Paris,* CXVIII (Paris 1894), 233–38.

rilaterals of latitude and longitude based on the ellipsoids of the *Carte de France*, Bessel and Clarke were used in combination with linear recording planimeters. The original copperplate map engravings were measured instead of paper map sheets, and each planimeter was equipped with a magnifying glass giving a twentyfold enlargement of its tracer point.

The men and institutions described in this chapter laid the foundations for modern European measurement of geographic area. They set the pace and provided the scientific methods and means for area measurement in all countries of Europe and in the remainder of the world. Since the contributions described, no fundamentally new or basicly improved area measurement undertakings have been recorded in the literature for Europe. The work of determining official geographic areas, without exception, became the responsibility of governmental bureaus. Many measurements were carried through after the first World War, for Europe changed from an area having 29 independent political units to one with a total of 39. New area measurements were abundantly recorded in official census and other statistical releases for each country and its subdivisions. Reference to the methods employed and the persons who directed the work are substantially lacking.[75]

[75] Tamss, F. "Die Bevölkerung der Erde," XIV, *Petermanns Mitteilungen*, Ergänzungsheft Nr. 212 (1931), 164 pages. This latest edition of the series provides an excellent statistical abstract for the Europe of post World War I. It should be noted that the work of Walther Schmiedeberg and Th. Willers. summarizing the European development, appropriately came in 1906 and 1911 after all of the principal contributions had been made.

Chapter VII. Developments in the United States

IN the United States, as elsewhere, the work of measuring geographic area had to await the construction of accurate maps. Developments in map making were retarded by vast stretches of unexplored land. Only through decades of effort by the U. S. General Land Office, the U. S. Geological Survey, the U. S. Coast and Geodetic Survey, and the United States Army did a fairly accurate and detailed cartographic picture evolve.[76] Aside from measurements for the areas of the thirteen original states, probably made by private citizens or by European geographers and appearing in American and foreign atlases and gazeteers,[77] there were no areas published by Federal agencies until fragmentary tables covering the acreage of public lands surveyed and sold began to receive periodic attention in the annual reports of the U. S. General Land Office.[78]

In the annual report for 1850, for the first time, total areas were given for the 17 States and 3 Territories being surveyed by the General Land Office. These totals were subdivided by the number of acres surveyed and unsurveyed.[79] No indication is given of the methods used in obtaining these areas. It would appear certain, however, that for the surveyed portions they resulted from summing acreages recorded on the original plat maps. As for the unsurveyed areas one may assume that they represent quotations from available geographic sources, making deductions for known areas.

The next contribution to the measurement of area in the United States was made for the Census of 1850 by Col. J. J. Abert of the Topographic Engineers of the United States Army. His work is dated September 25, 1853. Colonel Abert gave the area of the United States and its Territories as 2,983,153 square miles. He divided this total into separate areas for the Pacific Slope, the Mississippi Valley, and the Atlantic Slope. Further interesting comparisons were made between the large size of the United States, the smaller size of the principal European States, and the comparable size of the Roman Empire and of the area conquered by Alexander the Great.[80]

The first area table covering the entire United States by States and Territories was released in 1860 by the General Land Office. Here a total of 3,010,370 square miles is divided under the headings, States of the Union, Land States, District of Columbia, and Territories Embracing Public Lands. Further, it is stated, "to which added water surface, lakes, rivers, etc., we have a surface of over 3,250,000 square miles." [81] For the next two decades, the General Land Office remained the primary governmental source for the areas of all States and Territories. Their tables increased in scope to include the area of Alaska, to give historical annotations relative to the increase and decrease in the areas of certain States, and, for the first time, to give the areas of the counties of one State, namely Nebraska.[82]

Henry Gannett, Geographer for the Tenth Census on loan from the U. S. Geological Survey, laid the foundation for accurate and detailed area measurement in the

[76] Bowie, *op. cit.*, 348–356; Stewart, L. O.: *Public Land Surveys: History, Instructions and Methods* (Ames, Iowa: Collegiate Press, Inc., 1935), particularly pages 35–75; *Americana Yearbook, 1940* (New York, Americana Corporation, 1940), 330; Schley, J. L., "Some Giants of the Corps of Engineers," *Military Engineer*, (May–June, 1938); and other voluminous historical notes and descriptions in the *Military Engineer.*

[77] The first reference to the areas of various states of the United States uncovered by this investigation is given by Thomas Templeman in 1729. (See footnote 23 and Appendix A.)

A number of atlases published in this country prior to 1850 were examined: the Carey atlases of 1802 and 1810, the Carey and Lea atlas of 1823, the Tanner atlas of 1823, the Morse atlas of 1823, the Finley atlas of 1826, the Williams atlas of 1832, the Burr atlas of 1839 and the Mitchell atlases of 1846 and 1851. Of these only the Carey and Lea atlas of 1823 and the Finley atlas of 1826 gave areas for certain states, but no references were made as to source. After 1850, for the remainder of the nineteenth century, areas likewise were rarely given, and then only with an occasional vague statement of appreciation to the General Land Office or the Bureau of the Census.

[78] *U. S. American State Papers, Public Lands*, Vols. I to V (1789–1827), and in the bound volumes of the *Annual Report of the Commissioner of the General Land Office*, 1828 to the present. These reports, for the most part, were likewise printed in the *Congressional Record.*

[79] 31st Cong., 2d sess., Ex. Doc. No. 2, p. 2 (see Appendix B). Although this is believed to be the first comprehensive reference to the areas of all of the several States and Territories under the jurisdiction of the General Land Office, the areas of individual States obviously were ascertained earlier judging by the completion or virtual completion of the surveys in Alabama, Arkansas, Illinois, Indiana, Missouri, and Ohio prior to 1850. For a time table of the progress made in Public Land Surveys, see Stewart: *op. cit.*, 73.

[80] *Seventh Census of the United States, 1850.* (Washington: Robert Armstrong, Public Printer, 1853), XXIX, (see Appendix C).

[81] *Annual Report of the Commissioner of the General Land Office*, Washington, 1860, 13, (see Appendix D).

[82] *Ibid.*, (1870), 508. Other evidence of American progress in the measurement of area made between 1860 and 1880 can be traced in the following references: *Ibid.*, (1869), 225; *Ibid.*, (1870), 406 and 504; *Ibid.*, (1871), 297 and 340; *Ibid.*, (1875), 377 and 428; *Ibid.*, (1876), 186; *Ibid.*, (1877), 216; *Ibid.*, (1880), 271. An interesting summary table was prepared for the Census of 1870, citing the General Land Office as the source for United States area statistics. *The Statistics of the Population of the United States*, Ninth Census, Volume I, (Washington: Government Printing Office, 1872), 594 and 595 (see Appendix E).

United States. His publication of 1881, "The Areas of the United States, the Several States and Territories and Their Counties," [83] marks the beginning of six decades of work by the Census Office and the Bureau of the Census in supplying area figures on a county basis. In another way, the work of Gannett represents a landmark. For the first time an account is given of the methods and maps employed, the water bodies included, and the outer limits of the United States used as a basis for measurement. The following quotations summarize the nature of this work:

> The necessity for revising the figures which have popularly passed as the areas of the States presented itself early in the progress of . . . geographical work in connection with the Census. Of several States a number of estimates of area have been in use, differing from one another by thousands of square miles, and none of them, perhaps, traceable to any authentic source. Many of the results were palpably wrong, being so far from the truth that it is a source of surprise that they were not corrected before. . .
>
> The question, What constitutes the area of the United States? is by no means a simple one. Our jurisdiction extends to a line three nautical miles from the shore, but this strip of water cannot properly be regarded as a portion of the country. Supposing our boundary to be restricted to the sea-coast, there remains a question regarding the bays and estuaries of the sea. To what extent should the coast line be followed strictly, and where should we begin to jump across the indentations made by the sea? In this matter one can only follow his own judgment, making in each case as natural a decision as possible, as no criterion whatever can be established. . .
>
> Taking first the United States as a whole, its area was determined by summing up the square degrees. . . . Along the coast and the boundary lines of the country the areas of the fractional square degrees were obtained by direct measurement by means of scales, in every case measuring the smaller portion, whether land or water, to avoid, as far as possible, the errors incident to the projections and scales of the maps.
>
> The Atlantic, Pacific, and Gulf coasts were measured on charts of the Coast Survey; the shore lines of the great lakes on those of the United States Lake Survey; and the boundary with Mexico on the maps of the Mexican Boundary Commission. The northern boundary follows the parallel of 49° to the Lake of the Woods. Thence to Lake Superior is a weak place where we are dependent upon the map of Hind's exploration. . . . A second weak spot exists in the eastern boundary of Maine, from Schoodie Lake to the Atlantic coast. . . .
>
> The areas of the square degrees were computed by the . . . formula, derived by Mr. F. De Y. Carpenter.
>
> The areas of the several States . . . were determined in a similar manner.
>
> The areas of counties . . . are the results of direct measurement from the best county maps, corrected arbitrarily in order to make them add up to the areas of the States. In the States surveyed by the General Land Office, in which county lines follow the boundaries of survey townships, the areas (are of a high) degree of accuracy. In other States . . . the county lines are located by minor natural or artificial points . . . the probabilities are against their being correctly drawn on even the best maps, and the areas are correspondingly uncertain.
>
> The areas of included bodies of water, as bays and other inlets of the sea, lakes, ponds, etc., were, in most cases, determined by direct measurement from maps.
>
> The area of river surface has proved a very difficult matter to determine, even approximately.

HENRY GANNETT

Considering the current status of the map record in 1880 Gannett's work was of an exceptionally high order of accuracy; [84] with only minor alterations made to conform to obvious State and county boundary changes, his areas were retained for nearly sixty years. Alterations, restatements, and additions made by Gannett, acting as Geographer of the Census Bureau, can be traced by reference to Census publications released in 1891 and 1901.[85] As the result of a carefully prepared area table released by the General Land Office in 1899,[86]

[83] Gannett, Henry. *The Areas of the United States, the Several States and Territories and Their Counties,* Extra Census Bulletin, (Washington, Government Printing Office, 1881), 1–20. (See Appendix F for a facsimile reproduction of the entire Bulletin with the exception of the county area tables.)

For a short biographical summary of Henry Gannett's career (1846–1914) see: North, S. N. D. *Henry Gannett, President of the National Geographic Society, 1910–14.* (Washington: The National Geographic Society, 1915), 34 pages, from which the accompanying photographic portrait is reproduced.

[84] Undoubtedly polar planimeters were used to measure the irregular boundary portions which did not conform to one degree quadrilaterals of latitude and longitude although Gannett neglects to clarify this point. This assumption appears reasonable since already in 1877 a Boston firm was manufacturing and distributing these instruments. Herschel, Clemens: *Directions for Using the Planimeter as Made by Buff & Berger* (Boston: Press of Rockwell and Churchill, 1877).

[85] Gannett, Henry, *Areas of States and Counties,* Census Bulletin No. 23, January 21, 1891, (Washington: Government Printing Office, 1891), 1–14; and *Area in Square Miles of States, Territories and Counties,* Census Bulletin No. 57, February 23, 1901, (Washington: Government Printing Office, 1901), 1–11.

[86] *Annual Report of the Commissioner of the General Land Office, 1899,* (Washington: Government Printing Office, 1899), 86.

at some variance with Gannett's figures of 1901, collaboration was undertaken by Henry Gannett, then acting for the U. S. Geological Survey, C. S. Sloane, Geographer of the Census, and Frank Bond, Chief Draftsman of the General Land Office, to reconcile these differences and publish a joint statement. The result of this collaboration was published by Henry Gannett in 1906.[87] In harmony with the State totals thus agreed upon, revised land areas for the counties of the United States were published for the next three decennial censuses in 1911, 1921, and 1931.[88] The areas for the first two of these censuses were prepared under the supervision of C. S. Sloane, and those of the latter under the supervision of C. E. Batschelet, the present Geographer of the Bureau of the Census.

[87] Gannett, Henry, *The Areas of the United States, The States, and The Territories*, U. S. Geological Survey Bulletin No. 302, Series F, Geography, 53, (Washington: Government Printing Office, 1906), 1-9. The differences between the area tables of 1899 and 1901 were, for the most part, well within the limits of error for planimeter measurement. Certain variations, however, were considerable. These resulted from differences in the maps used and the inclusion in one case and the exclusion in another of certain embayments and coastal waters. Here the areas of the Great Lakes are given and credited to the several adjoining States, (see Appendix G).

[88] *Thirteenth Census of the United States, 1910.* Vol. I, Population, (Washington: Government Printing Office, 1911); *Fourteenth Census of the United States, 1920.* Vol. I, Population (Washington: Government Printing Office, 1921); *Fifteenth Census of the United States, 1930.* Vol. I, Population, (Washington: Government Printing Office, 1931).

Chapter VIII. Remeasurement of the United States: 1940

FOR decades, in connection with each decennial census of the United States, population returns have been published for the minor civil divisions of counties.[89] Areas for use in computing square mile density figures from census data, however, have only been provided on a county basis since Henry Gannett's publication of 1881. This was continued as a regular census practice for the next five decades. The next related advance was the publication by the Census Bureau in 1934 of State outline maps showing the minor civil divisions of counties. These maps were prepared under the supervision of C. E. Batschelet. The wide use made of these maps for plotting 1930 census statistics stimulated an active demand on the Bureau of the Census to undertake the measurement of the areas of these minor civil divisions.[90] However, measuring 51,600 minor civil divisions was an assignment requiring a considerable outlay of funds and a large group of workers to assure completion within practical time limits. The year 1937 proved to be most favorable for initiating this undertaking in the Bureau of the Census. Sufficient time remained before the Sixteenth Decennial Census to complete the work. Emergency funds and workers were available through the Work Projects Administration.

A map appraisal in the light of approved procedures for measuring geographic area was the first consideration. Maps were grouped into four categories of coverage: United States maps; State maps; county maps; and city maps. Since only the most accurate maps were to be used, consideration in virtually every case narrowed to one or at the most several map sources.

For complete coverage of the United States, the latest map series on one projection, and with the largest scale, proved to be the aeronautical charts prepared in 1937 by the U. S. Coast and Geodetic Survey. These charts, on a scale of 1: 500,000, are in quadrangle form covering two degrees of latitude and six degrees of longitude; are based on the Lambert conformal conic projection with standard parallels at 33° and 45°; are subdivided by a grid of 30 minute quadrilaterals of latitude and longitude; and possess reliable State boundary determinations. In fact, these charts, based on over 5 years of field and office research, represented such an improvement over all previous maps of this type that from the first there was no question as to the desirability of their use.

For State maps showing counties those released by the U. S. Geological Survey were selected. These maps, covering individual States on a scale of 1: 500,000 were based on the Lambert conformal conic projection and on a modified polyconic projection. Since they were printed over a period of three decades they naturally varied greatly in the correctness of their county boundaries. No better maps of this type were available, however, except for a recent compilation of Texas on a scale of 1: 750,000 prepared by the Post Office Department.[91]

For decades, the best county maps available had been collected in preparation for each census. The minor civil division boundaries, as well as the county boundaries, had been ascertained by correspondence with county officials. The function of conducting censuses

[89] Population returns have been published for the minor civil divisions or towns of the New England Sates since the First Census of 1790. This form of reporting, which was left in part to the discretion of the United States marshals conducting the census in each State, was not extended to include all States until the Tenth Decennial Census of 1880. Agricultural returns for minor civil divisions were not made available until 1930.

[90] Besides numerous verbal requests made by geographers and others that the Census Bureau undertake the measurement of minor civil division areas, this interest took definite form in the following published measurements: "Wisconsin Civil Township Land Areas Survey of the Acreages in Civil Towns 1935," *The Wisconsin Blue Book 1935*. Compiled by the Wisconsin Legislative Reference Library. (Madison, Wisconsin: Democrat Printing Co., 1935), pp. 672–680; Marshner, F. J., "Extent of Land and Water Area in Delaware and in Kent County," *An Economic Study of Land Utilization in Kent County, Delaware*, by R. O. Bausman, Bulletin No. 224, University of Delaware Agricultural Experiment Station, Newark, Delaware, (April, 1940), 34–38; Marschner, F. J. and Bausman, R. O., "Land and Water Areas of Delaware," *Delaware Notes, Twelfth Series, 1939*, University of Delaware, Newark, Delaware. Minor civil divisions areas were also measured and released in manuscript form by the U. S. Bureau of Agricultural Economics, and several State and local agencies for the following States or their subdivisions: Colorado, Georgia, Massachusetts, New Hampshire, New York, Pennsylvania, Rhode Island, Virginia, West Virginia, Wisconsin, and Wyoming; and parts of the States of Alabama, Kentucky, Maryland, Mississippi, North Carolina, South Carolina, Tennessee, and Washington.

[91] Other State maps showing counties, prepared by the General Land Office and the Post Office Department, on scales of 1: 760,320 and 1: 750,000, respectively, were not used either because of their age or smaller scale. The State transportation maps, on a scale of 1: 250,000, released by the Public Roads Administration and compiled and printed by the U. S. Geological Survey were not used because an excessive boundary line width precluded accurate planimeter measurement. The county boundary lines appearing on all maps were checked and when necessary corrected to conform to their status at the beginning date set for the enumeration of the Sixteenth Decennial Census.

had resulted in the Bureau of the Census being the one agency in the United States with a substantially complete minor civil division map record. As a prelude to area measurement, therefore, decisions had already been made concerning the relative accuracy and suitability of existing county maps.

The county maps used varied in scale from 1: 62,500 to 1: 125,000 and consisted of seven types: highway and road maps prepared by State highway departments in cooperation with the U. S. Public Roads Administration; topographic quadrangle sheets prepared by the U. S. Geological Survey; soil maps prepared by the U. S. Bureau of Agricultural Chemistry and Engineering; post route maps prepared by the Post Office Department; road maps prepared by county organizations or engineers, or by private surveyors; plat maps prepared by the General Land Office; and aerial photographs compiled into county maps by local associations cooperating with the Agricultural Adjustment Administration or the Soil Conservation Service. Of these seven types, highway and road maps were available for approximately 2,000 counties. For the remaining 1,000 counties the majority were covered by topographic quadrangle sheets and post route maps, whereas the other map types were used for only a few counties.

For several decades a collection had been made of city maps for all places of 2,500 or more inhabitants. Certified boundaries had been obtained from city officials. These maps were prepared by city engineers and private surveyors. They varied in scale from 1: 5,000 to 1: 24,000, and provided an accurate large-scale base for area measurement. In addition, through correspondence with city engineers the Census Bureau had obtained certified land and water areas for all cities having 2,500 or more inhabitants.

Before measuring the areas of counties, minor civil divisions and cities, it was necessary to decide on the outer limits of the United States and to establish mutually exclusive definitions for land and water. Setting outer limits for the United States required definite criteria for the inclusion or exclusion of bays, sounds, estuaries, and the Great Lakes. Decisions were needed relative to the width of streams to be measured as water or to be included as land; the size of lakes or ponds to be measured as water; the size of islands to be measured as land; and the treatment of river flood plains, swamps, tidal flats and other surfaces temporarily covered with water. For all of these decisions definite criteria were needed to minimize subjective choice.

The knottiest problem was that of setting outer limits for the United States. There were insistent demands for the inclusion of the Great Lakes in the areas of their adjoining States.[92] There were questions relative to the inclusion or exclusion of Long Island Sound, Delaware or Chesapeake Bays, the Straits of Juan de Fuca and Georgia, and coastal waters to a 3-mile or some other legal limit. Since the areas of several of the large coastal embayments and of the Great Lakes to the Canadian boundary were legally accredited to the adjoining States, it seemed obvious that these water areas required remeasurement. However, a definite rule for the manner of their inclusion or exclusion was lacking. Gannett in 1881 included Chesapeake Bay, but excluded Delaware Bay, Long Island Sound, the Great Lakes, and the Straits of Juan de Fuca and Georgia. At later dates Delaware Bay and the Great Lakes were included in a footnote to the land and inland water areas of the States, whereas the other areas continued to be excluded.[93]

This method of presentation was retained until the Census of 1940, when changes were made following extended discussion with cartographers, geographers, geodesists of the Federal map making agencies, and private scientific organizations. A quotation from the report of a special committee of the National Research Council dated May 3, 1941, is significant:

> In the interest of keeping our statistics of area upon the basis usual in foreign countries it was concluded that the Great Lakes areas, Long Island Sound, Delaware Bay, Chesapeake Bay, Puget Sound, and the Straits of Juan de Fuca and Georgia, etc., should be excluded from the inland waters in the main table but presented in the footnotes with the water areas of the several States.

The Canadian Census includes Canada's share of the Great Lakes in the total area of Ontario. Nevertheless, in Europe, where much of the principal work of area measurement has been done, the Caspian, Aral, and White Seas, and the Sea of Azov are not included in the total area of Russia; the various portions of the Baltic Sea are not in the totals for Germany, Denmark, Sweden, and Finland; the Sea of Marmara is not included in the total area of Turkey; and the Mediterranean and Black Seas are not treated as inland water, although they are certainly inland, or landlocked.

In the latest remeasurement of the United States, three fundamental definitions for land, inland water, and water other than inland water, were established for the first time. Since the Census Bureau is primarily a statistical agency, which collects, analyzes, and publishes statistics pertaining almost without exception, to

[92] Osborne, Chase S., and Stellanova, *Errors in Official U. S. Area Figures* (Lancaster, Pa.: The Science Press Printing Co., 1945).

[93] Gannett, H., *op. cit.*, 7 and 8; Bond, F., *Historical Sketch of "Louisiana" and the Louisiana Purchase*, Department of Interior, General Land Office, (Washington: Government Printing Office, 1933), 14.

land areas, area figures are given to enable users of statistical data to compute densities for comparison. From a statistical standpoint, small bodies of inland water are analogous to land area, but large water bodies are excluded from inland water, in order for densities not to be misleading.

A solution for the problem of setting outer limits for the United States was obtained by special adaptations, pertaining to embayments and islands, of the excellent principles established by S. W. Boggs, Geographer of the Department of State, in delimiting the territorial waters of the United States.[94] These adaptations of Boggs' principles resulted in the following rules for delimiting coastal and Great Lakes water,[95] and thereby, in part, for setting the outer water limits of the United States (fig. 8): (1) where the coast line is regular it shall be followed directly unless there are off-shore islands within ten nautical miles; [96] (2) where embayments occur having headlands of less than ten and more than one nautical mile in width, a straight line connecting the headlands shall set the limits; [97] however, (3) the coast line shall be followed if the indentation of the embayment is so shallow that its water area is less than the area of a semicircle drawn using the said straight line as a diameter; [98] and (4) two or more islands less than ten and more than one nautical mile from shore shall be connected by a straight line or lines, and other straight lines shall be drawn to the shore from the nearest point on each end island.[99]

Problems pertaining to the treatment of inland water required definitions in harmony with those established for coastal and Great Lakes water and for land. Such definitions were necessary since the outer limits of inland water are either conterminous with the inner limits of coastal or Great Lakes water, with the outer limits of the United States, or with the limits of land. To meet these problems for inland water the same 4 rules were used, but a limit of 1 nautical mile was substituted for the 10 nautical mile limit. In addition, inland water was defined to include: permanent inland water surface, such as lakes, reservoirs and ponds having 40 acres or more of area; streams, sloughs, estuaries, and canals one-eighth of a statute mile or more in width; and islands having less than 40 acres of area.

Besides setting outer limits for the United States and distinguishing between inland water and the water of coastal embayments and the Great Lakes, the following definitions were established for land. Land was defined to include: dry land and land temporarily or partially covered by water, such as marshland, swamps, and river flood plains; [100] streams, sloughs, estuaries, and canals less than one-eighth of a statute mile in width; and lakes, reservoirs and ponds having less than 40 acres of area.[101]

After selecting the best maps, setting outer limits for the United States, and distinguishing between land and water, every effort was expended to select the most reliable area measurement method compatible with the necessarily miscellaneous character of the best maps available, the character of the labor supply, and the magnitude of the undertaking in view of required completion dates. There was little question regarding the ideal area measurement method. Such a method would have used geodetic area tables for 5-minute or even smaller quadrilaterals of latitude and longitude, a polar planimeter to measure the boundary fragments, one thoroughly trained technician to do all the original work, a second equally competent technician to repeat each operation as a check, and a third such person to remeasure and correct all errors. Wide, but necessary departures were made from this impractical ideal procedure in view of the magnitude of the assignment.[102]

As a practical expedient, it was decided that the area of the United States and of the individual States should be determined from the single best map base giving Nation-wide coverage on one scale and one projection. This was done using the U. S. Coast and Geodetic Aeronautical Charts (1: 500,000) of 1937, geodetic area tables for 30-minute quadrilaterals of latitude and longitude based on the Clarke (1866) dimensions adjusted to the legal meter, and planimeter measurements of those quadrilaterals cut by boundary lines. It was further decided that the areas of the counties of each State, as determined by planimeter measurement, must be adjusted to equal the predetermined State totals. Finally, it was decided that the areas of minor civil

[94] Boggs, S. W., "Delimitation of the Territorial Sea, The Method of Delimitation Proposed by the Delegation of the United States at the Hague Conference for the Codification of International Law," *American Journal of International Law*, Vol. 24 (July, 1930), 541–555.

[95] Termed "State water" and not subdivided among adjoining counties or minor civil divisions.

[96] See Atlantic coastal strip, C. of fig. 8., plate I.

[97] See Atlantic coastal strips A.. and B. of fig. 8., plate I.

[98] See Pacific coastal strips H., J., K. and L. of fig. 8., plates XII and XIII.

[99] See treatment of Florida Keys, Atlantic coastal strip Q. of fig. 8.

[100] It was decided to omit tidal flats, even though these appear on pilot charts and some of the older U. S. G. S. topographic quadrangles, and to use the coast line (mean high tide) shown on the maps selected for area measurement.

[101] These definitions and those for inland water are in terms of practical limits for planimeter measurement when working on maps with scales varying from 1:62,500 to 1:125,000.

[102] For an example and full explanation of this painstaking procedure applied to topographic quadrangles covering the small State of Delaware see Marschner and Bausman, *op. cit.* The tabulations, in manuscript form, showing the areas of various cities and towns in the State of Massachusetts, prepared by the Commonwealth of Massachusetts Department of Public Works, 100 Nashua Street, Boston, 1938, apparently afford another example, although a full explanation of the procedure is not available.

divisions of each county as determined by planimeter measurement, must be adjusted to equal the predetermined county totals. These adjustments to the State areas were considered necessary to avoid the confusion of a total area for the United States based in part on county and minor civil division areas obtained from geodetic area tables for 5-minute squares and planimeter measurement of U. S. G. S. topographic quadrangle sheets and in part on less reliable areas obtained by simply planimetering highly miscellaneous maps. The desirability of using topographic quadrangle sheets to obtain areas of lasting value for each county so covered was advocated by F. J. Marschner,[103] and is to be strongly seconded. This procedure, however, is desirable only if one wishes to obtain the areas of counties and States covered by topographic quadrangle sheets. It is not feasible when one has the objective of obtaining complete and comparable coverage for the entire United States.

The fallacy inherent in obtaining a total area for the United States by summing the areas of all minor civil divisions as obtained by planimeter measurement from miscellaneous large-scale maps, is the certainty of duplication and omission in the map coverage. If a given county is accurately covered by topographic quadrangle sheets, there is no assurance that the boundaries of the adjoining counties, which should match exactly, do so when shown on other less accurate maps. There is an unaccountable and disturbing variation in the placement of parallels and meridians, and in some instances in their complete omission. A combined total of areas containing these inaccuracies might be negatively or positively cumulative or essentially compensatory. Without question, the total area would combine a complex hodgepodge of inaccurate and accurate measurement. The results would be at variance with the total area of the United States or with the area of the individual States as could be more reliably determined by a combined use of geodetic area tables and planimeter measurements from maps on a single smaller scale, based on one projection, and covering the entire area.[104]

[103] Marschner, F. J., *The Need of Revision of Areal Figures of the United States by Counties With Tables of the Areas of Quadrilaterals of the Earth's Surface of Five Minutes Extent in Latitude and Longitude.* U. S. Department of Agriculture, Washington, D. C., October 1929.

[104] This assumption was substantiated. The United States total obtained by summing the unadjusted minor civil division areas of all States was 3,002,158 square miles, or 229 square miles less than the total obtained by measuring the 1: 500,000 U. S. Coast and Geodetic aeronautical charts. However, the process of summing the totals for individual States had a compensatory result, judging by the wide variation among the individual State totals. For Florida and Montana alone the totals were +1,184 and −1,627 square miles, respectively; 16 other States showed plus or minus variations of between 100 and 500 square miles; and for the remaining 30 States only 6 had variations of less than 10 square miles. As would be expected these variations were lowest for those States completely covered by topographic quadrangle sheets.

Provisions were made to safeguard the accuracy of all measurements and computations involved in obtaining the minor civil division, county, State and United States areas. Each operation was done in original by a force of workers and repeated in independent duplicate by a second force. Finally the results were compared in an independent check by a third force. Faulty operations were repeated until satisfactory. In planimeter measurement a maximum variation of .60 percent was allowed between the original and duplicate readings. Planimeter operators were required to encircle each area three times and take three consecutive readings before raising the tracer needles of their instruments. Planimeters were tested and adjusted at regular intervals. These safeguards were used to insure the accuracy of the results even under the mass operations required to complete the assignment.[105]

Not all minor civil divisions were measured by planimeter. Large portions of the country having essentially rectangle minor civil divisions, covered by the township and range surveys of the U. S. General Land Office, were scaled by ruler. This procedure obtained accurate areas at a great saving of time. Every effort was made to assure complete coverage for the water area of all reservoirs. Reservoir records were ob-

[105] This project employed upwards of 100 people for three years. At its operation peak a total of 118 planimeter operators were used and the work was completed at a cost of approximately $300,000.

The following table shows the limits of variation allowed between original and duplicate planimeter readings for various map scales . . . Proportional limits of variation were allowed for other map scales. All planimeters were calibrated to read in hundredths of square inches. Areas of over 25 square inches were further subdivided before being measured in order to avoid the errors which resulted from extending these essentially standard small-sized planimeters too close to their capacities. Figure 7 shows the type of planimeter used.

Established Limits of Variation in Planimeter Readings

Area in square inches	Limit of variation in square inches	Limit of variation in percent
	Map scale of 1:62,500	
To 5.00	0.03	0.60
5.00 to 10.00	.05	.50
10.00 to 15.00	.08	.53
15.00 to 25.00	.10	.40
	Map scale of 1:125,000	
To 5.00	0.02	.40
5.00 to 10.00	.04	.40
10.00 to 15.00	.06	.40
15.00 to 25.00	.08	.32
	Map scale of 1:250,000	
To 5.00	0.01	.20
5.00 to 10.00	.03	.30
10.00 to 15.00	.05	.33
15.00 to 25.00	.06	.24
	Map scale of 1:500,000	
To 5.00	0.01	.20
5.00 to 10.00	.02	.20
10.00 to 15.00	.03	.20
15.00 to 25.00	.04	.16

tained from the Soil Conservation Service, the Federal Power Commission, the U. S. Bureau of Reclamation, the Tennessee Valley Authority, the Army Engineer Corps, and from several State governments. These were checked to the county maps used in measuring the minor civil divisions. Reservoirs not completed by April 1, 1940, were ignored for measurement since the entire map record was prepared to fit this starting date of the Sixteenth Decennial Census. Precautions were taken to check the minor civil division areas against all comparable area records available for any portion of the country.[106] Wide variations were carefully rechecked and corrected if found in error. County areas published for the Census of 1930 were compared with the county areas computed for 1940. County areas for six Midwestern States were obtained by summing the acreage data on the plat maps of the U. S. General Land Office and these areas were used as an additional check.[107] Special precautions were taken to insure accurate city areas. When more than a 3-percent variation existed between the Census area for a city and the area claimed by the city engineer the difference was reconciled by correspondence or when this was not possible the Census area was taken. When the official city area varied less than 3-percent from the Census area, it was accepted and the necessary adjustments were made in the adjoining minor civil divisions.

The newly measured minor civil division areas and the remeasured areas of the counties and States of the United States were published in book form accompanied by revised State outline maps showing minor civil division boundaries.[108] These minor civil division area and map data provide the foundation for a diverse array of density distribution maps based on 1940 Census statistics. Such density distribution maps possess 16 times the areal refinement provided on a county basis. Immediate use has been made of these minor civil division areas in preparing a population density map of the United States showing density gradations in color.[109] Other work of a similar nature undoubtedly will be undertaken by governmental and private research workers.

One very important source of error is eliminated by the use of geodetic area tables, namely, the changes in area resulting from the expansion and contraction of map paper with changes in atmospheric humidity. Since quadrilaterals of latitude and longitude expand and contract with the map paper and since any measurements made within the quadrilateral are always adjusted to a fixed value, this variable effect is nullified. See Th. Willers for a discussion of this and other means for reducing errors in the measurement of geographic area.

[106] See footnote 90 for the minor civil division area sources used as a check.

[107] The use of the U. S. General Land Office plat maps as a means for obtaining the areas of minor civil divisions was abandoned because of the profound changes which had occurred in the water surfaces shown on these maps. This condition is to be expected since many of the surveys which produced these plat maps were completed during the first half of the nineteenth century.

[108] Batschelet, C. E. and Proudfoot, M. J., *Areas of the United States: 1940*, Sixteenth Census of the United States (Washington: Government Printing Office, 1942) (see Appendix H).

[109] This work, in wall map form, on a scale of 1:2,500,000 was brought to completion by C. E. Batschelet with the initial collaboration of M. J. Proudfoot.

Figure 8

THE OUTER LIMITS OF THE UNITED STATES AS ESTABLISHED FOR "THE AREAS OF THE UNITED STATES: 1940"

This multisectional strip map, consisting of Plates I to XIV, traces the outer limits of the United States. These limits were established by definition for the remeasurement of the "Areas of the United States: 1940," a special publication of the Sixteenth Decennial Census. The limits may be followed from match line to match line from the Canadian-Maine border to the Texas-Mexican border; from the Washington-Canadian border to the Mexican-California border; and then again along the Great Lakes boundary line between Canada and the several adjoining States. The cross-hatched area was defined as "state water" and credited to the several adjoining States (appendix H). The water area inland from the State water was defined as "inland water." Not only was the inland water credited to the adjoining States, but it was further subdivided among the adjoining counties and the minor civil divisions of the counties.

Thus by following a prescribed set of definitions for State and inland water, the limits established could be used again, or duplicated for a check of the results. The application of a uniform set of definitions, if they were accorded international acceptance, might result in desirable uniformity in the future measurement of geographic area.

Plate I

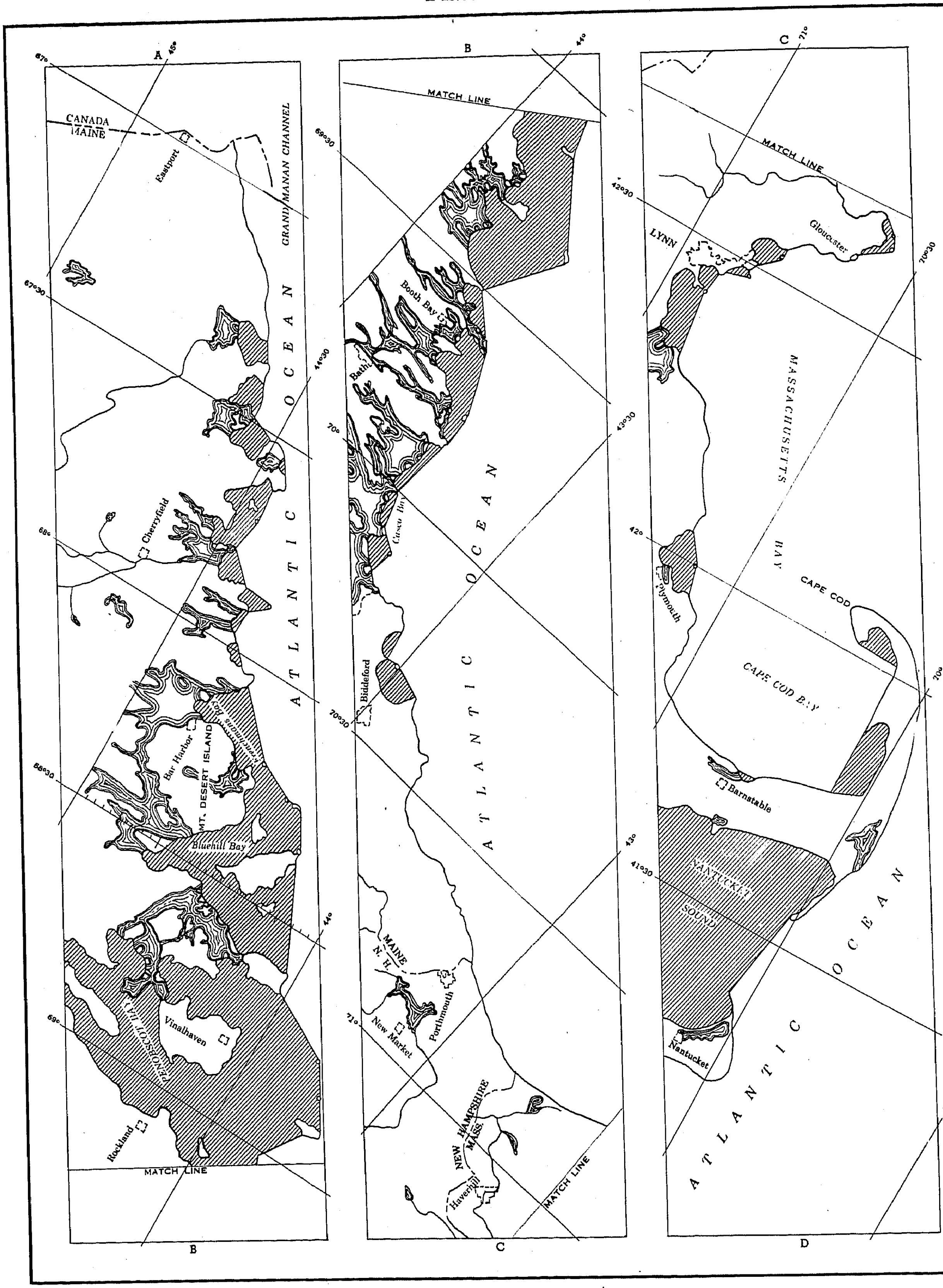

A
B
C
D
MATCH LINE
CANADA
MAINE
Eastport
GRAND MANAN CHANNEL
ATLANTIC OCEAN
Cherryfield
Bar Harbor
MT. DESERT ISLAND
Bluehill Bay
PENOBSCOT BAY
Vinalhaven
Rockland
Booth Bay
Bath
Casco Bay
Biddeford
MAINE
N. H.
Porthmouth
New Market
NEW HAMPSHIRE
MASS.
Haverhill
LYNN
Gloucester
MASSACHUSETTS BAY
Plymouth
CAPE COD
CAPE COD BAY
Barnstable
NANTUCKET SOUND
Nantucket

Plate II

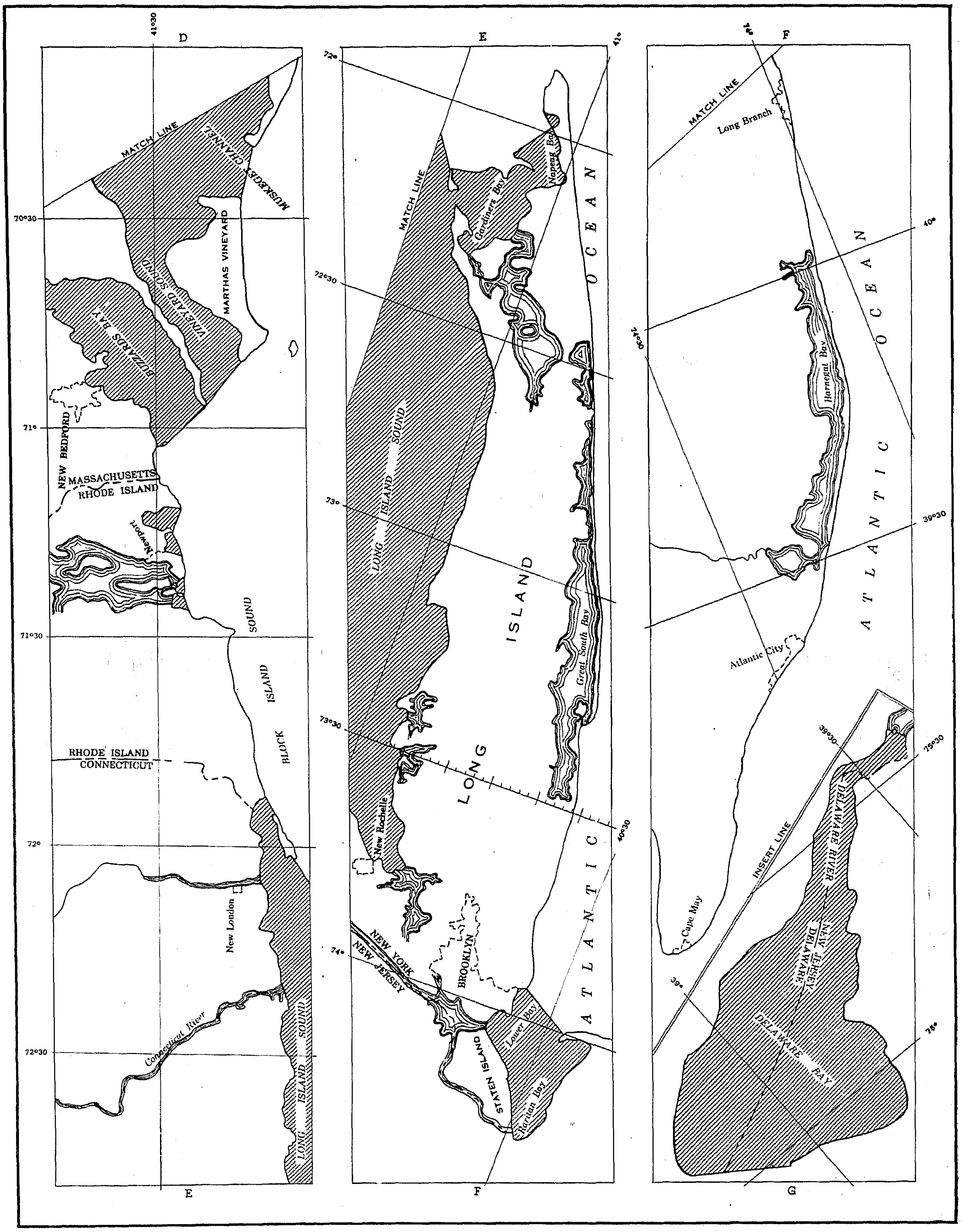

D
E
F
G
MATCH LINE
MUSKEGET CHANNEL
VINEYARD SOUND
MARTHAS VINEYARD
BUZZARDS BAY
NEW BEDFORD
MASSACHUSETTS
RHODE ISLAND
Newport
BLOCK ISLAND SOUND
RHODE ISLAND
CONNECTICUT
New London
Connecticut River
LONG ISLAND SOUND
Gardiners Bay
Napeag Bay
LONG ISLAND
Great South Bay
New Rochelle
NEW YORK
NEW JERSEY
BROOKLYN
STATEN ISLAND
Lower Bay
Raritan Bay
ATLANTIC OCEAN
Long Branch
Barnegat Bay
Atlantic City
Cape May
INSERT LINE
DELAWARE RIVER
NEW JERSEY
DELAWARE
DELAWARE BAY
41°30
70°30
71°
71°30
72°
72°30
73°
73°30
74°
41°
40°30
40°
39°30
39°
75°
75°30

Plate V


M
81°30
MATCH LINE
31°30
Brunswick
31°
GEORGIA
FLORIDA
ATLANTIC OCEAN
30°30
JACKSONVILLE
30°
St. Augustine
N
81°30
MATCH LINE
29°30
81°
OCEAN
Daytona
Daytona Beach
29°
ATLANTIC
RIVER
28°30
ST. JOHNS
80°30
28°
O
80°30
MATCH LINE
RIVER
Vero Beach
INDIAN
OCEAN
27°30
Fort Pierce
ATLANTIC
27°
80°
West Palm Beach
Palm Beach
Lake Worth
26°30
Fort Lauderdale
P

Plate VI

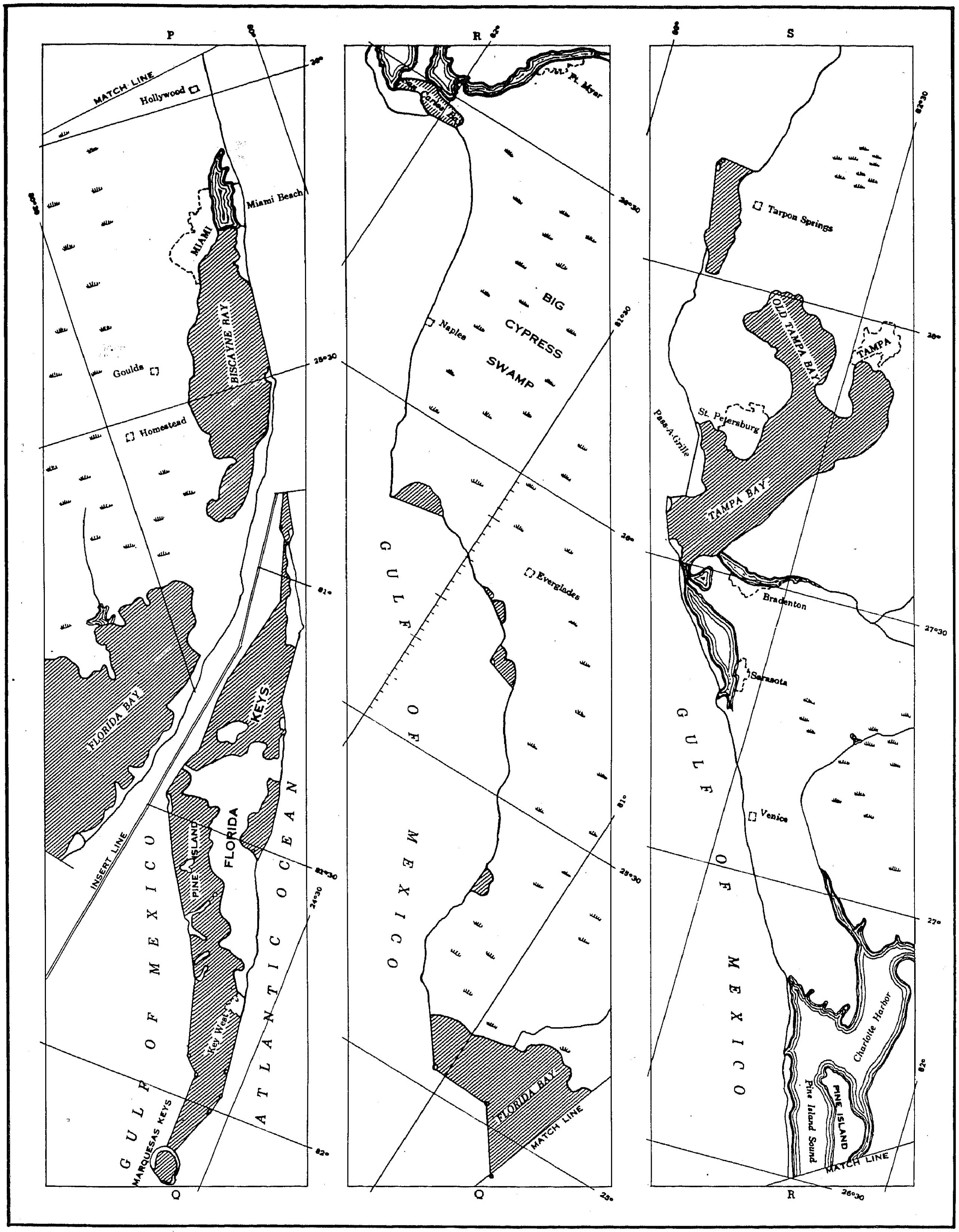

P
R
S
Q
Q
R
MATCH LINE
Hollywood
Miami Beach
MIAMI
BISCAYNE BAY
Goulds
Homestead
FLORIDA BAY
KEYS
FLORIDA
PINE ISLAND
Key West
MARQUESAS KEYS
INSERT LINE
GULF OF MEXICO
ATLANTIC OCEAN
Ft. Myer
Naples
BIG CYPRESS SWAMP
Everglades
GULF OF MEXICO
FLORIDA BAY
MATCH LINE
Tarpon Springs
OLD TAMPA BAY
TAMPA
St. Petersburg
Pass-A-Grille
TAMPA BAY
Bradenton
Sarasota
Venice
GULF OF MEXICO
Charlotte Harbor
PINE ISLAND
Pine Island Sound
MATCH LINE
26°30
26°
25°30
25°
27°30
27°
28°
28°30
26°30
81°
81°30
82°
82°30

Plate VII

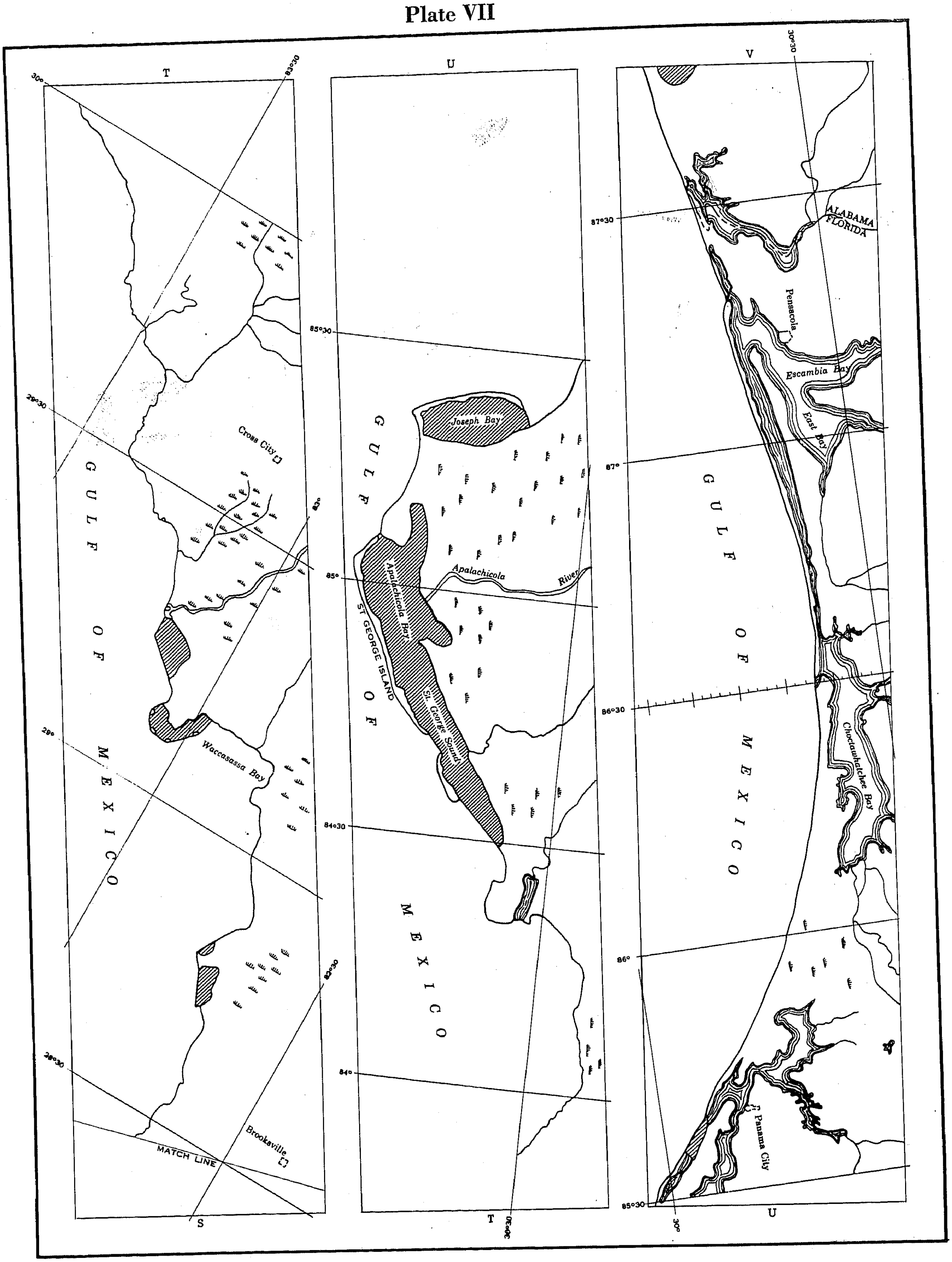

T
U
V
30°
83°30
29°30
Cross City
GULF OF MEXICO
83°
29°
Waccasassa Bay
28°30
82°30
Brooksville
MATCH LINE
S
85°30
Joseph Bay
Apalachicola Bay
Apalachicola
River
85°
ST GEORGE ISLAND
St. George Sound
84°30
84°
T
30°30
87°30
ALABAMA
FLORIDA
Pensacola
Escambia Bay
East Bay
87°
86°30
Choctawhatchee Bay
86°
Panama City
85°30
U

Plate VIII

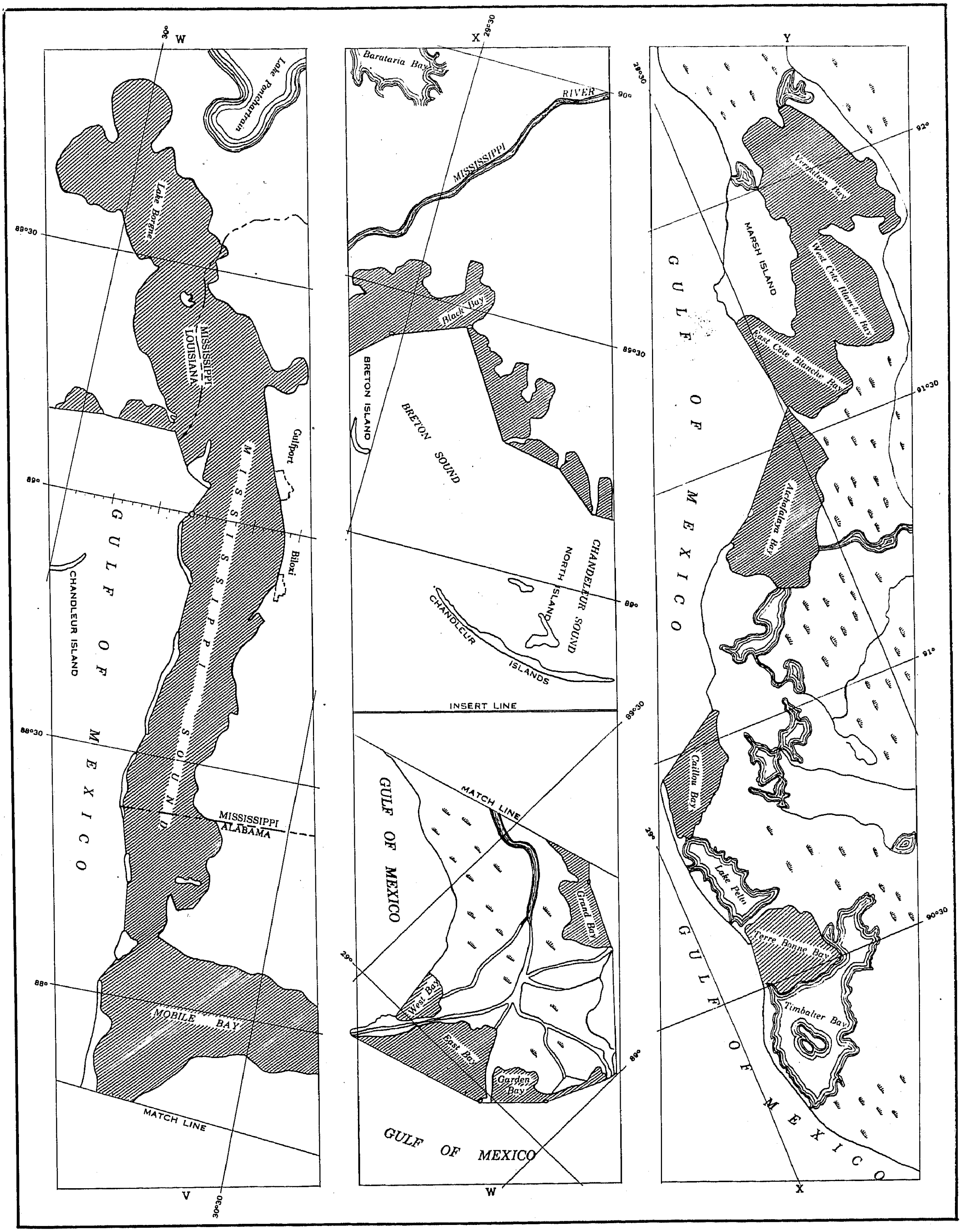

Lake Pontchartrain
Lake Borgne
MISSISSIPPI
LOUISIANA
MISSISSIPPI SOUND
Gulfport
Biloxi
GULF OF MEXICO
CHANDLEUR ISLAND
MISSISSIPPI
ALABAMA
MOBILE BAY
MATCH LINE
Barataria Bay
MISSISSIPPI
RIVER
Black Bay
BRETON ISLAND
BRETON SOUND
CHANDELEUR SOUND
NORTH ISLAND
CHANDLEUR ISLANDS
INSERT LINE
GULF OF MEXICO
MATCH LINE
Grand Bay
West Bay
East Bay
Garden Bay
GULF OF MEXICO
Vermilion Bay
MARSH ISLAND
West Cote Blanche Bay
East Cote Blanche Bay
GULF OF MEXICO
Atchafalaya Bay
Caillou Bay
Lake Pelto
Terre Bonne Bay
Timbalier Bay
GULF OF MEXICO
W
V
X
Y
30°
30°30
29°30
89°30
89°
88°30
88°
90°
29°
92°
91°30
91°
90°30

Plate IX

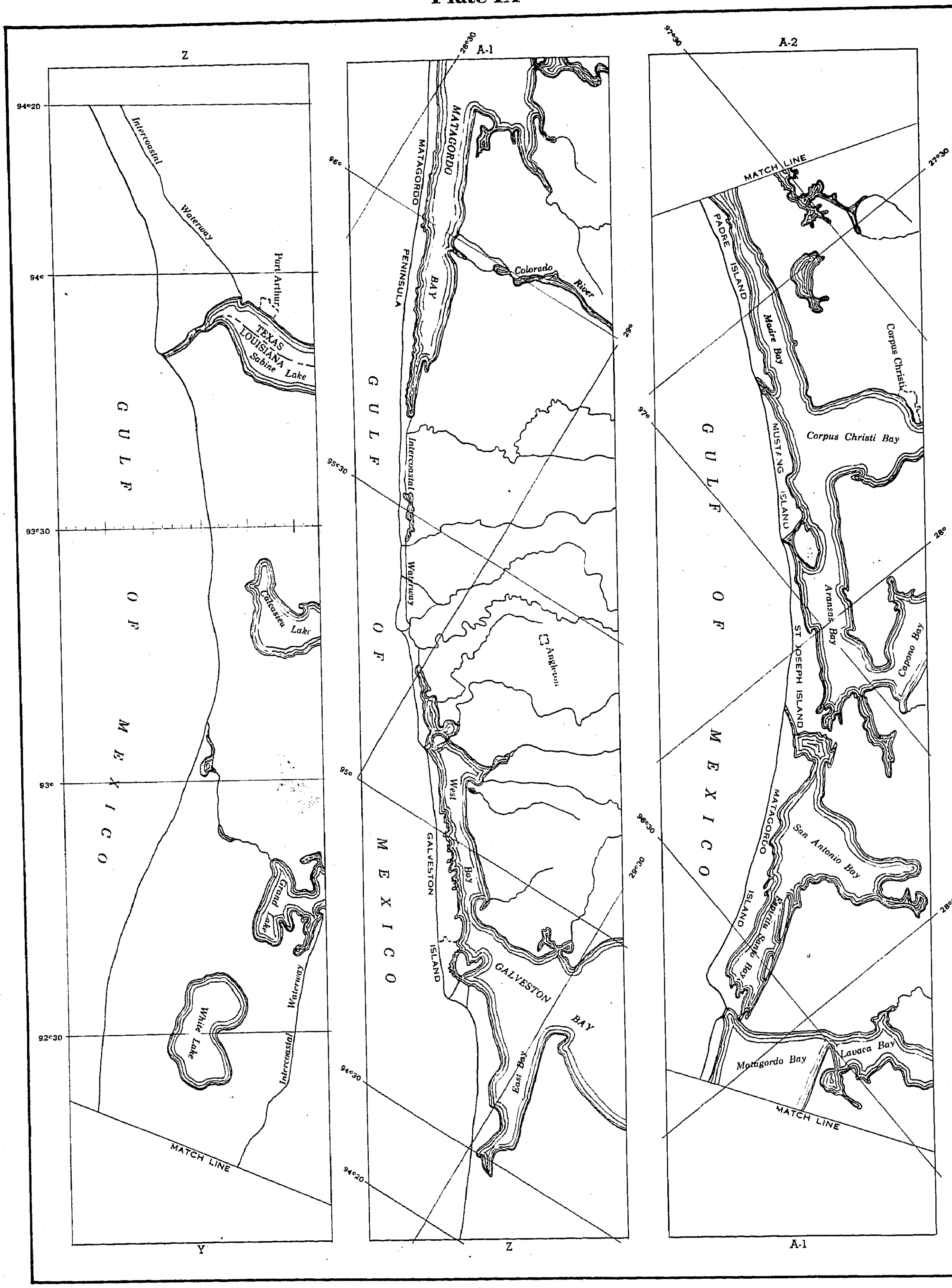

Z
94°20
94°
93°30
93°
92°30
Intercoastal Waterway
Port Arthur
TEXAS
LOUISIANA
Sabine Lake
Calcasieu Lake
Grand Lake
White Lake
Intercoastal Waterway
GULF OF MEXICO
MATCH LINE
Y
A-1
28°30
96°
95°30
95°
94°30
94°20
29°
29°30
MATAGORDO
MATAGORDO PENINSULA
BAY
Colorado River
Intercoastal Waterway
Angleton
West Bay
GALVESTON ISLAND
GALVESTON BAY
East Bay
GULF OF MEXICO
Z
A-2
97°30
27°30
97°
96°30
28°
28°30
MATCH LINE
PADRE ISLAND
Madre Bay
Corpus Christi
Corpus Christi Bay
MUSTANG ISLAND
Aransas Bay
Copano Bay
ST JOSEPH ISLAND
MATAGORDO ISLAND
San Antonio Bay
Espiritu Santo Bay
Matagordo Bay
Lavaca Bay
GULF OF MEXICO
MATCH LINE
A-1

Plate X

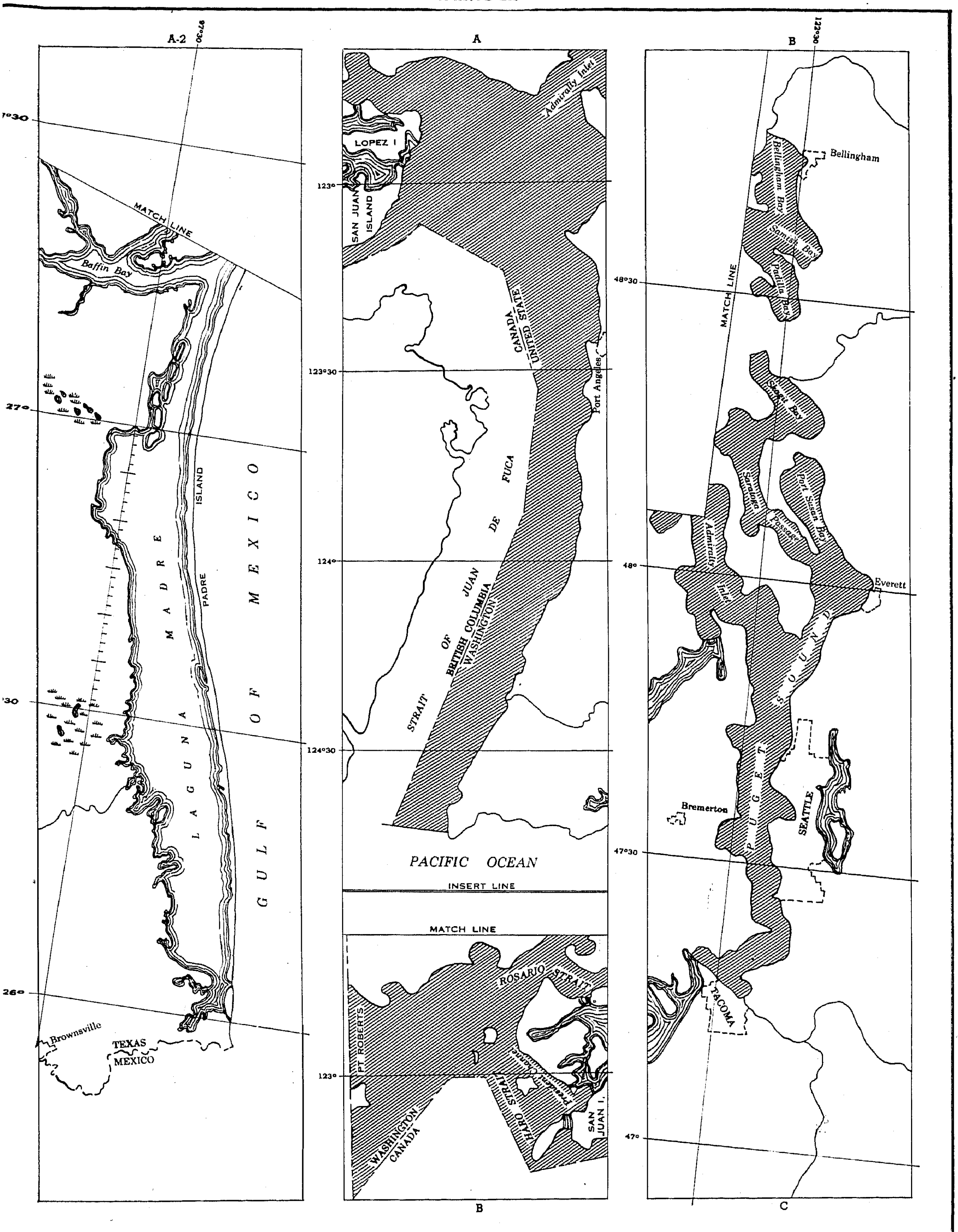

A-2
A
B
C
MATCH LINE
INSERT LINE
Baffin Bay
LAGUNA MADRE
PADRE ISLAND
GULF OF MEXICO
Brownsville
TEXAS
MEXICO
LOPEZ I
SAN JUAN ISLAND
Admiralty Inlet
CANADA
UNITED STATE
Port Angeles
STRAIT OF JUAN DE FUCA
BRITISH COLUMBIA
WASHINGTON
PACIFIC OCEAN
ROSARIO STRAIT
PT. ROBERTS
HARO STRAIT
SAN JUAN I.
CANADA
Bellingham
Bellingham Bay
Samish Bay
Padilla Bay
Skagit Bay
Saratoga
Port Susan Bay
Admiralty Inlet
Everett
PUGET SOUND
Bremerton
SEATTLE
TACOMA
48°30
48°
47°30
47°
123°
123°30
124°
124°30
26°
27°

Plate XI

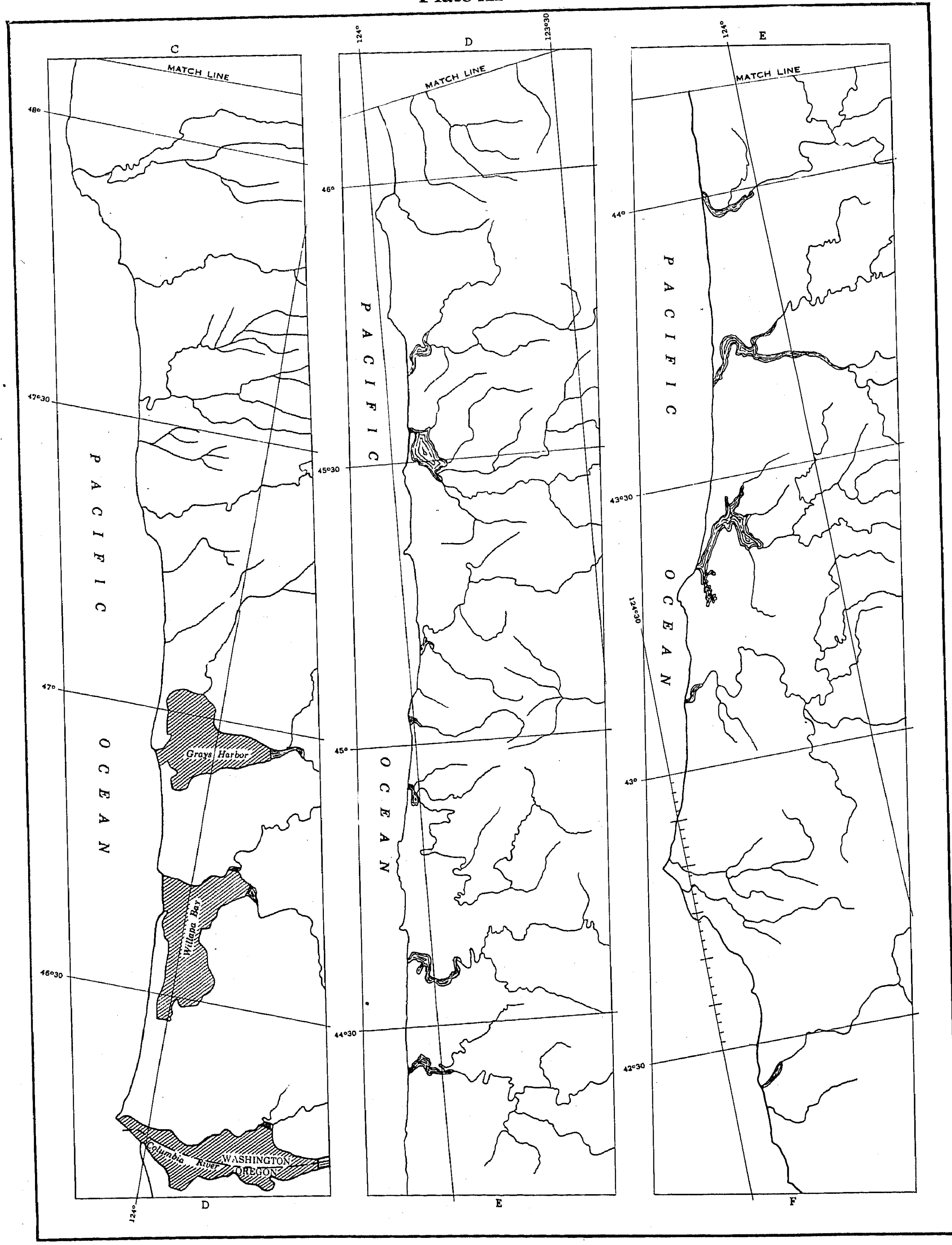

C
MATCH LINE
48°
47°30
P A C I F I C
47°
O C E A N
Grays Harbor
Willapa Bay
46°30
Columbia River
WASHINGTON
OREGON
124°
D
124°
D
123°30
MATCH LINE
46°
P A C I F I C
45°30
45°
O C E A N
44°30
E
124°
E
MATCH LINE
44°
P A C I F I C
43°30
O C E A N
124°30
43°
42°30
F

Plate XII

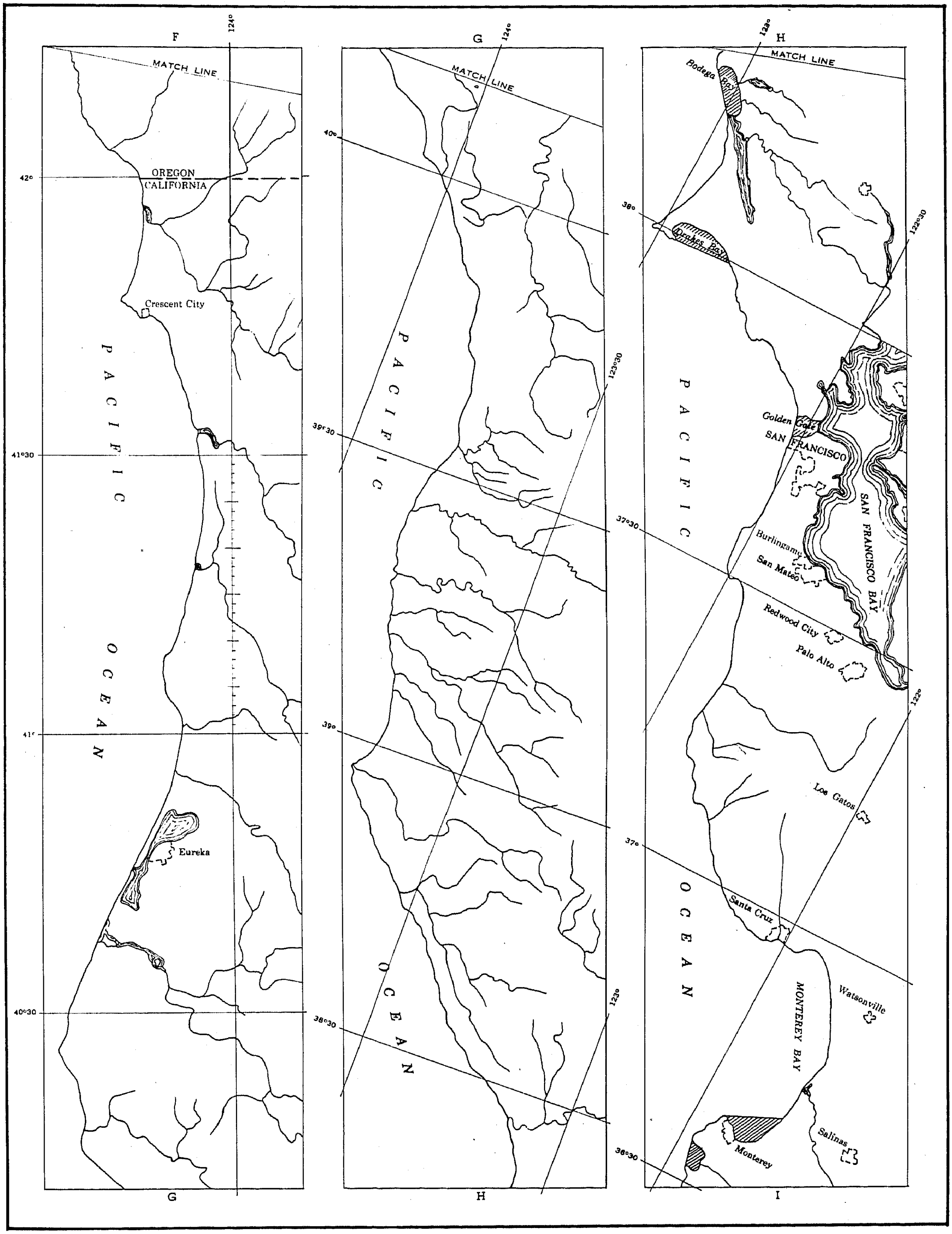

F
124°
MATCH LINE
42°
OREGON
CALIFORNIA
Crescent City
P A C I F I C
41°30
O C E A N
41°
Eureka
40°30
G
124°
MATCH LINE
40°
P A C I F I C
123°30
39°30
39°
O C E A N
38°30
123°
H
123°
H
MATCH LINE
Bodega Bay
38°
Drakes Bay
122°30
P A C I F I C
Golden Gate
SAN FRANCISCO
SAN FRANCISCO BAY
37°30
Burlingame
San Mateo
Redwood City
Palo Alto
122°
Los Gatos
37°
O C E A N
Santa Cruz
Watsonville
MONTEREY BAY
Salinas
Monterey
36°30
I

Plate XIII

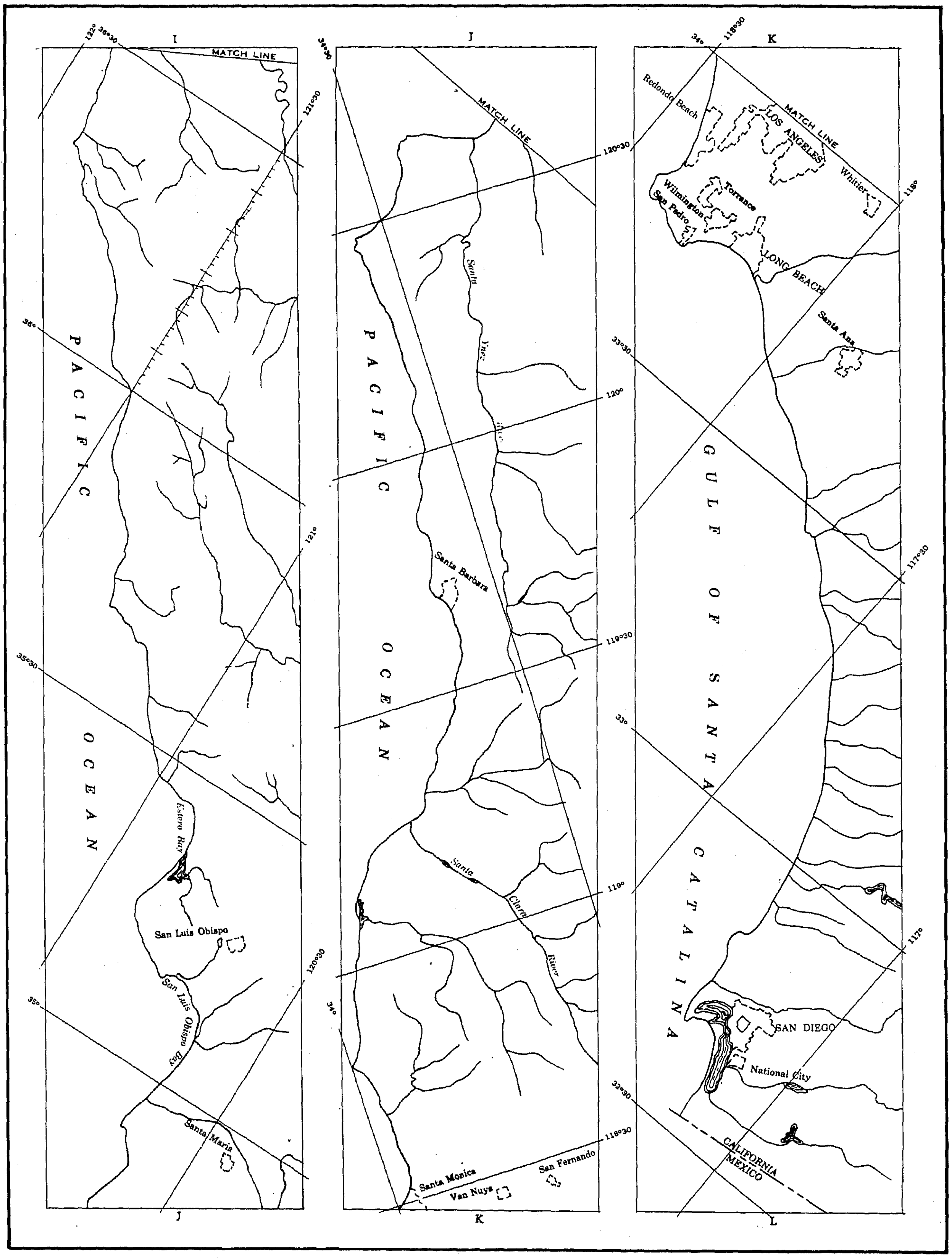
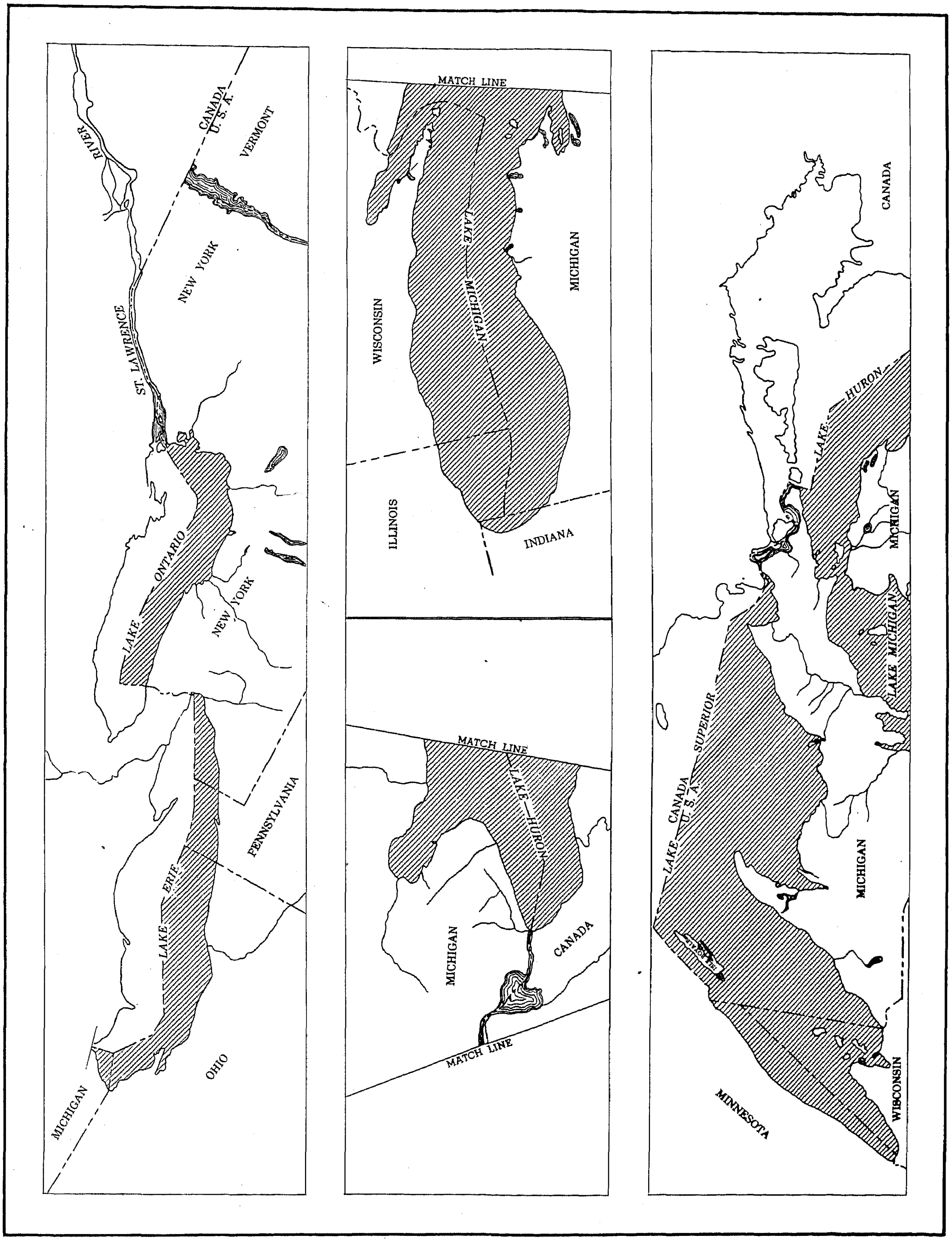

I
J
K
MATCH LINE
PACIFIC OCEAN
GULF OF SANTA CATALINA
122°
35°30
36°
121°30
121°
120°30
35°
34°30
120°30
120°
119°30
119°
118°30
34°
33°30
33°
32°30
118°
117°30
117°
Estero Bay
San Luis Obispo
San Luis Obispo Bay
Santa Maria
Santa Ynez River
Santa Barbara
Santa Clara River
Santa Monica
Van Nuys
San Fernando
Redondo Beach
LOS ANGELES
Whittier
Torrance
Wilmington
San Pedro
LONG BEACH
Santa Ana
SAN DIEGO
National City
CALIFORNIA
MEXICO
L

Plate XIV


CANADA
U. S. A.
VERMONT
RIVER
NEW YORK
ST. LAWRENCE
LAKE ONTARIO
NEW YORK
PENNSYLVANIA
LAKE ERIE
OHIO
MICHIGAN
MATCH LINE
WISCONSIN
LAKE MICHIGAN
MICHIGAN
ILLINOIS
INDIANA
MATCH LINE
LAKE HURON
MICHIGAN
CANADA
MATCH LINE
CANADA
LAKE HURON
MICHIGAN
LAKE MICHIGAN
LAKE SUPERIOR
CANADA
U. S. A.
MICHIGAN
MINNESOTA
WISCONSIN