Chapter 1--INTRODUCTION, BACKGROUND, AND SUMMARY

This report, one of a series of evaluation and research reports providing measures of the quality of the 1960 Censuses of Population and Housing, is concerned with the contribution of interviewers and crew leaders to the total mean-square error of census statistics.

Project A of the 1960 Evaluation and Research Program was composed of three studies to measure the variances in census statistics introduced by interviewers. crew leaders, respondents, and coders. (Other projects in this program have been reported in Evaluation and Research Program reports, series ER 60, Nos. 1-6.) The largest study in this group was Response Variance Study I (referred to subsequently in this report as the RVS) to measure the response variance due to interviewers and crew leaders. Also included in Project A are Response Variance Studies II and III to measure the contribution of respondents and coders, respectively, to the errors of census statistics. These latter studies will be discussed in another report.

There are several reasons to attempt to measure the response variance. What we would like to publish in the census volumes are the "true" values of population characteristics. What we do publish are estimates of these "true" values. At the Bureau of the Census, we have spent years devising methods to measure the distance from the published statistics to the "true" values. The work reported here is an attempt to measure part of this distance. We are concerned with the errors introduced into the data during the enumeration process. Specifically, in this study, we focus on the errors introduced by census crew leaders and interviewers.

After the sampling variance has been accounted for, the remainder of the difference between the "true" value and the published statistic is measured by a bias term. By means of the mathematical model described in chapter 4, we have been able to treat some bias errors as variances. The model treats the bias associated with a particular interviewer as a source of variability. We assume that each interviewer is selected from a universe of interviewers. Then, the constant tendency of an interviewer to overestimate or underestimate, his "personal equation", is a random event in a particular trial and can be treated as a source of variability. By this means we have been able to split out the crew leader and interviewer effect. Of course, any constant-error tendencies that all interviewers or crew leaders have in common are not reflected in a variance term. We deal only with those biases that vary from one interviewer or crew leader to another.

A second reason for instituting a large-scale experiment to measure response variance is that there was strong evidence from the 1950 census that the response variance due to census interviewers had a considerable effect on statistics for small areas. Self-enumeration was introduced on a large scale in the 1960 censuses to reduce the effect of interviewers. The RVS was incorporated into the evaluation program to find out if the use of self-enumeration was successful in reducing the interviewer component of the mean-square error.

The method used to estimate the interviewer component of the mean-square error was to compare the work of pairs of interviewers, each of whom had been randomly assigned to work in half of a selected geographic area. These estimates were averaged over all pairs in the study, so that averages of response variances are presented for areas of 3,900 persons for different items. The crew leaders also had an assignment pattern such that their effect could be measured. Estimates of average values of response variances are presented separately for highly urban, other urban, and rural areas.

There are six general points to be made in summarizing the results of this study.

1. The interviewers had an effect on the published statistics from the 1960 censuses. For some characteristics this effect was small, but for some it was very large. The 86 items studied were separated into two groups-those for which the response variance was more than half the size of the sampling variance, and those for which it was less. For the data which measured the interviewer effect only, 38 items were in the first group. Those items were as follows:

- 6 nonresponses from a total of 6
- 4 housing items from a total of 5
- 7 school enrollment items from a total of 10
- 4 educational attainment items from a total of 8
- 1 nativity item from a total of 2
- 5 other income items from a total of 11
- 5 wage and salary items from a total of 12
- 4 self-employment income items from a total of 11
- migration item from a total of 3 1
- 1 labor force item from a total of 6
- 0 number of children items from a total of 4
- 0 occupation items from a total of 7
- 0 veteran status items from a total of 1

Though the response variances were very high for nonresponse items, there were also other characteristics such as school enrollment, and housing items connected with gross rent that were very much affected by interviewers. Some characteristics such as number of children, occupation, and veteran status were affected very little. Excluding the nonresponse items, some of the largest ratios of response variance to sampling variance were found for the following items: native born, unemployed, highest grade of school attended not completed, and enrolled in the first year of college. The response variances for these items were much larger than the sampling variances. However, it should be kept in mind that for the majority of items the response variance was less than half the size of the sampling variance.

2. The crew leaders also had an effect on the published statistics from the 1960 census. The same kind of grouping was done for the 86 items from the data which measured the combined effect of the crew leaders and interviewers. Forty-five items were in the group for which the response variance was more than half the size of the sampling variance. Those items were as follows:

- 6 nonresponses from a total of 6
- 4 housing items from a total of 5
- 6 school enrollment items from a total of 10
- 5 educational attainment items from a total of 8
- 1 nativity item from a total of 2
- 6 other income items from a total of 11
- 5 wage and salary items from a total of 12
- 6 self-employment income items from a total of 11
- 3 migration items from a total of 3
- 2 labor force items from a total of 2
- 0 number of children items from a total of 4
- 1 occupation item from a total of 7
- 0 veteran status items from a total of 1

The kinds of items affected were about the same except that the migration items were very heavily affected by the crew leader. Again, such items as number of children, occupation, and veteran status were affected very little. The crew leader effect was small for most items. However, for at least 15 items, the crew leader effect was quite large and cannot be explained by the sampling variability of the estimate of response variance. 3. Though the interviewer had an effect on the 1960 census statistics, even with the use of self-enumeration, the response variances in 1960 were much smaller than those in 1950. As shown in chapter 6, the level of re-sponse variance in 1960 is about one-fourth of the 1950 level.

4. Estimates of the response variance were computed for highly urban, other urban, and rural areas. The response variance for a given statistic varied depending on the type of area. For example, the level of response variance over all wage and salary income items was largest in highly urban areas. However, the level for mobility items was largest in rural areas.

5. A small number of the total pairs of interviewers in the RVS accounted for large proportions of the response variance. This was true for each one of the items studied.

6. A small number of interviewers, about 10 percent, were consistently poor for many items but among the remainder most who performed poorly on one or two items did not do badly for the other items.

The general design of the study and the processing of the data are discussed in chapters 2 and 3. Chapters 4 and 5 deal with the underlying mathematical model and the distributions of the estimators. Chapters 6 and 7 contain the results of the study. Chapter 8 has a comparison of the RVS crew leaders and interviewers with regular census crew leaders and interviewers. Chapter 9 contains a discussion of the limitations of the data.

Chapter 2--DESCRIPTION OF THE 1960 CENSUS RESPONSE VARIANCE STUDY

The Response Variance Study was designed to measure the contribution of interviewers and crew leaders to the nonsampling variability of census data. This study relates only to the second stage of the census in the sample areas. A brief description of the two-stage census procedure follows. For a fuller description, see Bureau of the Census, 1960 Censuses of Population and Housing: Procedural History.

The enumeration districts (ED's) for the 1960 census had been laid out so that the boundaries of any of the statistical areas for which data were to be published were not crossed. Some ED's were too small or too sparsely populated to be an assignment for an interviewer. Often two or more ED's were combined into a single enumeration area (EA) which was the assignment for a single interviewer. There was a total of about 160,000 EA's, each with an estimated population of about 1,100 persons.

During the week preceding April 1, 1960, the Post Office distributed Advance Census Reports to all households. These reports were to be filled out by a member of the household and held for the visit of a census interviewer. This Stage I interviewer canvassed all households in the EA's assigned to him, made a complete listing of housing units in the EA's, collected the Advance Census Reports which contained limited data on population and housing characteristics, and transcribed these data to Stage I enumeration schedules which could be computer processed. In addition, he conducted interviews in those households where no Advance Census Report had been delivered or where it was not complete, and corrected omissions and wrong entries. The Stage I interviewer also left a Household Questionnaire at every fourth household asking for detailed information on population and housing characteristics. The head of the household was asked to fill out the questionnaire and mail it to the local census office within three days.

About one-third of the Stage I interviewers were employed for Stage II of the census. They were given the original listing of households made by the Stage I interviewer; that listing showed which households had been given the Household Questionnaires. The Stage II interviewers were to interview the households which had been requested to send in the questionnaires but had not done so. They also interviewed households from which defective or incomplete questionnaires had been received. They then transcribed these data to the Stage II enumeration schedules for computer processing. On a nationwide basis, about 45 percent of all Household Questionnaires were received complete, consistent, and requiring no followup.

The RVS was limited to Stage II of the census and thus to areas where a two-stage census procedure was to be used which comprised 82 percent of the total population. The remaining 18 percent of the population lived in sparsely populated or rural areas in which a onestage census was to be conducted. That part of the population was not included in this study.

In the two-stage areas, 292 census supervisory districts were established for the purpose of taking the census. Most of these supervisory districts corresponded in boundary with congressional districts. The remainder were formed either by combining or splitting congressional districts for census purposes. A district office might be one of several in a large city such as New York.

The 292 two-stage supervisory districts were arranged in 50 strata, each stratum containing about 2,800,000 persons. Each stratum was contained completely within one of the four geographic regions of the United States. A stratum might contain offices from different States, as long as the States were within the same region. For example, one stratum contained districts from Vermont, Massachusetts and Rhode Island. A selection of one supervisory district from each stratum was made, the selection based on probability proportionate to the estimated 1960 population contained within the district. Table 1 shows the 50 areas in which the RVS was conducted.

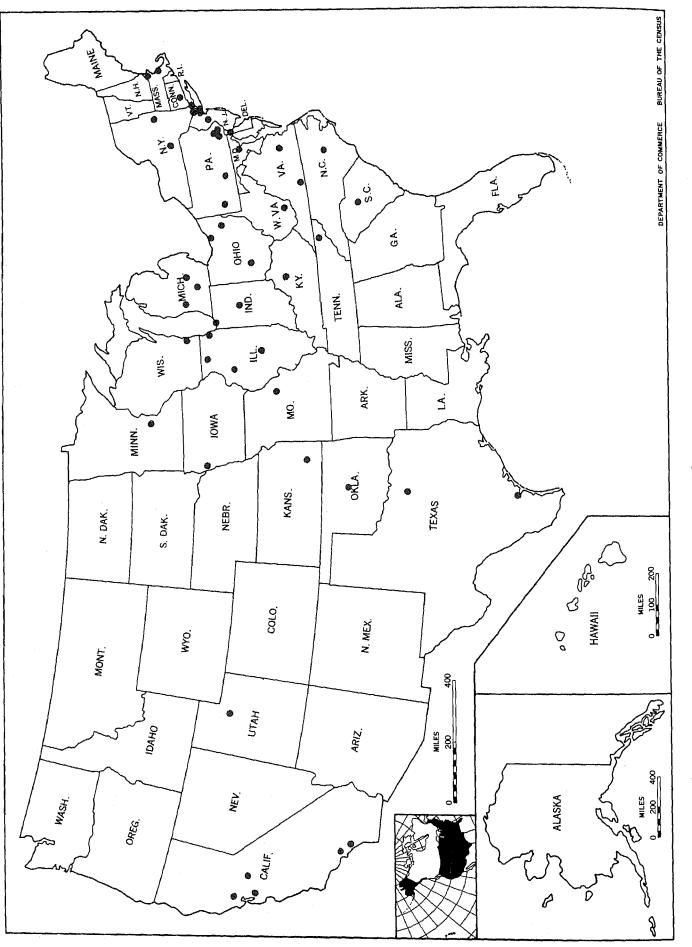
Table 1 shows that each of the four regions of the United States and 26 of the States were represented in the RVS. These sample areas are shown in their approximate locations on the map in figure 1.

Because the study areas were so scattered and because the regular census personnel were fully occupied with census duties, it was necessary to have a trained person in each area who would be responsible for the organization, supervision and control of the RVS. To meet this goal, over 50 statisticians were recruited from other government agencies, other survey organizations, and universities. These statisticians were designated as experimental program (EP) specialists.

The EP specialists were brought to Washington, D.C. a week before the beginning of Stage I of the census and given intensive training on census procedures and procedures for the conduct of the RVS. After the training sessions, one specialist was sent to each of the 50 sample areas. The New York, Chicago, and Elizabeth, N.J. offices had two specialists.

Further sampling was now necessary to reduce the size of the study area. Figure 2 may help clarify the sampling steps and the assignment of crew leaders and interviewers.

One of the first tasks of the EP specialist was to reduce the size of the study area by selecting a sample of crew leader districts. In order to do this, he was to form groups of contiguous Stage II crew leader districts.



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Figure 1. Approximate Locations of Response Variance Offices

Table 1.--AREAS SELECTED FOR RESPONSE VARIANCE STUDY

Region	State	Area Name
Northeast	Connecticut	New Haven
	Massachusetts	Salem Quincy
	New York	Jamaica Brooklyn New York (Manhattan) Bronx Troy Binghamton
	New Jersey	Elizabeth
	Pennsylvania	Philadelphia Philadelphia Chester Lewistown Pittsburgh
North Central	Michigan	Flint Battle Creek Grand Rapids
	Illinois	Peoria Chicago Decatur Rockford
	Wisconsin	Racine
	Ohio	Canton Cleveland Springfield
	Indiana	Kokomo Gary
	Iowa	Sioux City
	Minnesota	Minneapolis
	Kansas	Independence
	Missouri	Mexico
South	Maryland	Towson
	Delaware	Wilmington
	West Virginia	Bluefield
	Virginia	Richmond Danville
	North Carolina	Goldsboro
	South Carolina	Columbia
	Kentucky	Lexington
	Tennessee	Johnson City
	Oklahoma	Oklahoma City
	Texas	Corpus Christi Dallas
West	Utah	Salt Lake City
	California	Stockton Los Angeles Santa Ana Oakland Santa Rosa

The groups were usually pairs of crew leader districts, but in some instances were groups of three crew leader districts or a single crew leader district. Only crew leaders who used the same type of enumeration schedule (block or non-block) were to be put in the same group. (See Series ER60 No. 1, Evaluation and Research Program of the U.S. Censuses of Population and Housing, 1960: Background, Procedures, and Forms for samples of the forms used by the EP specialist in grouping the crew leader districts.) The total number of groups of crew leader districts varied from one sample area to the next, but was usually from five to seven. After all Stage II crew leader districts were grouped, the EP specialist selected a random number from a table of random numbers. This number corresponded to one of the groups of crew leader districts. This group was to be the reduced study area within the sample area. The steps the EP specialist took in selecting the group of crew leader districts were carefully documented and forms from each of the 50 sample areas were mailed to Washington where they were reviewed by the technical staff.

The EP specialist then obtained a list of all "special" enumeration districts (ED's) contained within the selected group of crew leader districts. These ED's--jails, hotels, dormitories, homes for the aged, and so forth--were excluded from the RVS. The EP specialist then went through the remaining EA's, recording the population count, if available, and determining ways to group four or five contiguous EA's into clusters. The EP specialist followed these general principles in creating the clusters:

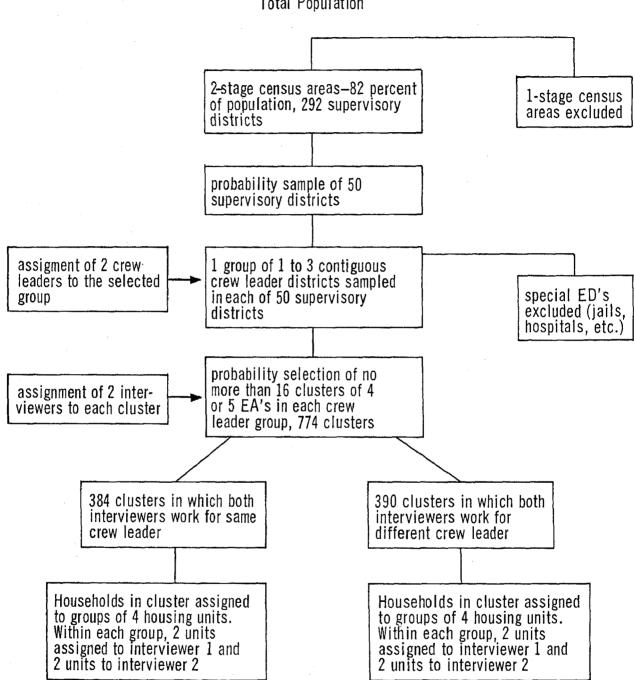
- 1. Form clusters of contiguous EA's.
- 2. Form clusters as compact as possible to reduce the amount of travel by interviewers.
- 3. Form clusters which are fairly uniform in population size.

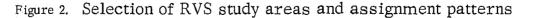
When all the EA's had been assigned to clusters, the EP specialist listed each cluster with its EA's and the EA populations on a designated form. If he had 16 clusters or less, all of the clusters were kept for the study. If he had more than 16 clusters, he selected 16 of them, by a probability method, to be in the study. This was the final stage of sampling in the RVS. The results were as follows:

	Number of clusters
Number of areas	of EA's
40	16
5	15
3	13
1	12
1	8

Thus, a total of 774 clusters of EA's were designated for the study. In each of these clusters, two interviewers were responsible for the collection of the Stage II census data, giving a total of 1,548 interviewers in the study. Also, in each sample area there were two crew leaders, or a total of 100 crew leaders in the study.

The EP specialist had the task of assigning interviewers and crew leaders to the clusters of EA's. He filled out a form (Form 60-28-16.7) listing the clusters and the EA numbers within clusters. From names supplied to him by the District Office staff, he wrote in the names of two interviewers next to each cluster. This was not a random assignment of any two interviewers in the whole area to the cluster. The EP specialist was given a list of interviewers selected by the District Supervisor to work in Stage II. He reviewed this list with the





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crew leaders, so that interviewers could be assigned to the cluster of EA's nearest their homes. The names of the two interviewers for a cluster were listed alphabetically. If the cluster was odd-numbered (determined by the order in which the clusters were copied to the form) each of the two interviewers was assigned to a different crew leader. If the cluster was even-numbered. both interviewers were assigned to the same crew leader. This method of assignment gave an interpenetrated pattern for interviewers and in half the clusters. an interpenetrated pattern for crew leaders. As it turned out, in 384 clusters both interviewers were assigned to the same crew leader. Estimates of the interviewer effect were obtained from these clusters. In 390 clusters. each interviewer was assigned to a different crew leader. From these clusters, estimates of the crew leader effect were added to the interviewer effect. (See Equation 4.25 shows the term chapter 4, page 13. which measures the crew leader effect.)

One of the most important tasks and one of the most difficult to organize remained for the EP specialist. This was the randomization of the housing units within clusters between the pairs of interviewers. Within every group of four housing units, two were to be assigned to one interviewer of the pair, and two to the other interviewer. (For a description of the process, see *Response Variance Study*, Manual for the Experimental Program Specialist, 1960 Census.)

Because two interviewers were working in the same EA's, many of the enumeration materials had to be duplicated. The listing books were copied, so that each interviewer could be provided with one. Maps for each EA were photocopied. Also, the original Stage II enumeration books prepared by the Stage I interviewers were transcribed into two new books. The EP specialist in each area was responsible for these tasks being correctly completed before the date scheduled for the start of the Stage II enumeration.

As far as possible, conditions in the study area were the same as the regular census conditions. A summary of the major differences from regular census procedures follows:

1. An EP specialist was designated to take charge of the RVS in each of the sample areas.

2. If there had been no RVS, each Stage II crew leader district would have had only one Stage II crew leader assigned to it. Because of the interpenetrated sample design of the RVS, two crew leaders worked in both of the two crew leader districts in the study for each area. However, each RVS crew leader was assigned about 16 interviewers, just as in the regular census.

3. In the census there were about 48 EA's in each Stage II crew leader district. In the RVS, there were about 32 EA's in each crew leader district.

4. In the non-RVS areas, one interviewer did all the work in each of three EA's. In RVS areas, one interviewer worked in half of four EA's, or an average of two EA's per interviewer.

5. The EP specialist gave additional training to the crew leaders assigned to the RVS area to explain the few changes in census procedures necessitated by the RVS, such as how to select pages for field review of the interviewer's work.

6. Duplicates of the listing book in all RVS EA's were prepared; maps for RVS EA's were photocopied; two RVS ED books were prepared, one for each interviewer, from the original Stage II book.

7. The EP specialist was responsible for making the random assignments of sample housing units to the pairs of interviewers.

Except for the differences listed, the census enumeration procedures were the same in the RVS areas. The same questionnaires were used, crew leaders and interviewers received the same training as in non-RVS areas, pay rates were the same, and the interviewers had the same job to do in the RVS areas as in the non-RVS areas.

Chapter 3--PROCESSING OF THE DATA

The processing of the data from the Response Variance Study was a very complicated task. As with the regular census materials, all RVS enumeration books were sent to the Bureau of the Census Operations Office in Jeffersonville, Ind., where all basic census processing was done.

The first major processing operation to which all of the census schedules were subjected was coding. Each entry written in by the interviewer on place of birth, migration, education, income, occupation, industry, and so forth, was converted to a numerical code. The RVS EA's went through the census coding and editing processes in the same way as all other EA's.

At the same time that the coding was being done, a critical review of the sample to detect biases in selection of households by size of household was being carried out for all census ED's. Only about one percent of the ED's required special handling to correct for the biases detected by this review. In those ED's, specified sample households were cancelled and others were duplicated. The RVS ED's were subjected to this review and in some ED's, households were cancelled and others duplicated as in the non-RVS ED's. Other than New York State, only 16 ED's were affected by the bias review. In these 16 ED's, 164 housing units were cancelled and 153 others were duplicated. Information on the number of ED's affected in New York was never made available. However, since there was a total of 4,560 ED's in the RVS, it seems likely that less than one percent were affected by the bias review.

Following this, all census schedules were microfilmed and converted to magnetic tapes for computer processing. Two standard census edits were performed on the data. A complicated allocation process was performed on the data. During the editing and allocation, inconsistencies between different items were eliminated and entries for nonresponses were imputed. The method of allocation of entries was by a so-called "hot deck" procedure. The last response for a person in a certain age, sex, color, etc. group was stored in the computer. A nonresponse for a person with similar characteristics was imputed that same value. The data presented in chapters 6 and 7 show results before and after allocation for nonresponses and inconsistencies. However, limits were established for the number of substitutions and allocations that were permitted without further investigation in an ED. (See Bureau of the Census, 1960 Censuses of Population and Housing: Procedural History, for a complete description of the census processing and allocation procedure.)

Because the RVS enumeration schedules were processed exactly as all other census schedules, some of the original differences between interviewers were eliminated. The processing described above had the effect of making the schedules more uniform. However, the procedure makes the measures of response variance in chapter 7 applicable to the published census statistics.

After the RVS EA's had been through all census processing, identification codes were added to the basic census record for each housing unit included in the RVS. These included the state code, ED number, listing book page and line number, ED book page number, interviewer number and group-of-four serial number. This group-offour serial number identified the basic group of four housing units in which two were assigned to one interviewer and two were assigned to the other interviewer. In addition, a code showing whether there had been any unusual occurrences such as a refusal followed by the enumeration of the household by the crew leader, the close-out of a unit because of inability to contact the householder after three calls, and other such things in the enumeration of the household, was made for every housing unit in the RVS. There were about 285,000 housing units for which these basic records were made. However, since there were many sources from which information on unusual enumerations were available, one household could have more than one record. For example, a close-out enumeration was detected from the listing book and a record made of that. The same household might have been cancelled during the bias review, so a record was made of that.

An identification record for RVS ED's was also made. This record showed the State, ED number, cluster number and any unusual events which had happened to the ED such as being rejected by the computer edit programs. These ED identification records were matched to the census tapes and then, on the basis of ED number and page number, the individual household records were matched to the basic census tape.

Following this, the census units were regrouped into their basic groups-of-four, as originally assigned. The computation of response variances was carried out using only complete groups of four housing units in which two of the housing units were assigned to one interviewer and the remaining two to the other interviewer. Incomplete groups-of-four were not included in the computation. Because of losses of units in incomplete groups-offour, losses of units where the identification records could not be matched to the basic census records, and losses of units due to the computer edits, the final data are based on approximately 247,000 households, a loss of 13 percent.

Table 2 shows a frequency distribution of the number of clusters by percent of units lost during processing. Those units which were lost during processing had no chance to be included in the computation of the response variance. Over half of the clusters of EA's lost less than 10 percent of the total number of units.

Percent of housing units	Clusters without crew leader effect	Clusters with crew leader effect	All clusters	
randomized but lost during processing			Number	Percent lost
0.0-4.9. 5.0-9.9. 10.0-14.9 15.0-19.9 20.0-24.9 25.0-29.9 30.0-34.9 35.0-39.9 40.0-44.9 45.0-49.9 50.0-59.9 60.0-69.9 70.0-79.9	77 129 77 37 21 12 12 12 7 3 4 5 0	71 132 66 42 31 14 10 9 5 4 4 4 1	148 261 143 79 52 26 22 16 8 8 9 1	19.1 33.7 18.5 10.2 6.7 3.4 2.8 2.1 1.0 1.0 1.0 1.2 .1
Total	384	390	774	99.9

Table 2 .-- NUMBER OF CLUSTERS BY PERCENT OF HOUSING UNITS ORIGINALLY RANDOMIZED BUT LOST DURING PROCESSING

The analysis of the RVS was done in two parts. The 384 clusters of EA's in which each interviewer was supervised by the same crew leader are referred to as the clusters without the crew leader effect. The data for these clusters are based on about 122,500 housing units and 370,000 persons. The 390 clusters of EA's in which each interviewer was supervised by a different crev leader are referred to as the clusters with the crew leader effect. The data for these clusters are based on approximately 124,500 housing units and 375,000 persons

Chapter 4--THE MATHEMATICAL MODEL

The mathematical model used in the Response Variance Study is the model described by Hansen, Hurwitz, and Bershad [6]. In this model, the term survey is used for either a census or a survey. The survey process is regarded as being repeatable. Each survey is regarded as one trial from among all possible repetitions of the survey under the same general conditions. These general conditions specify the questionnaire used, the training of the interviewers, the method of recording information, and so forth. The results of any one trial are assumed not to be influenced by any earlier trial.

The basic concepts necessary to the understanding of the mathematical model underlying the RVS were presented by Hansen, Hurwitz, and Madow [7]. If it were possible to interview each individual repeatedly, a population of responses for each individual would be generated. In a survey, then, we get a sample of respondents and thus a sample of possible responses from each of the sample persons.

Assuming that each survey is regarded as a trial and that the same general conditions hold for each trial, a random variable, x_{jt} , is defined to have the value 1 if the j-th unit has the characteristic of interest on the t-th trial and is defined to have the value 0 otherwise. An estimate of the proportion of the population having the characteristic is:

$$p_{t} = \frac{1}{n_{t}} \sum_{j}^{n_{t}} x_{jt}$$
(4.1)

where n_t is the number of units in the sample in trial t. We are interested in the variance of this estimate which is:

$$\sigma_{\mathrm{p}_{\mathrm{t}}}^{2} = \mathrm{E}(\mathrm{p}_{\mathrm{t}} - \mathrm{P})^{2} \qquad (4.2)$$

The expected value operation indicated above and in the further development, unless otherwise stated, represents the average value taken over all possible trials including all possible samples and all possible responses, and

$$P = E p_{+}$$

Suppose now, that the j-th unit is held fixed. Consider all possible repetitions of the measurements over all possible samples and trials on this j-th unit. Then, the conditional expected value over all such possible measurements is:

$$\mathbf{E}_{\mathbf{i}} \mathbf{x}_{\mathbf{it}} = \mathbf{P}_{\mathbf{i}} \tag{4.3}$$

It follows that $0 \le P_i \le 1$. Then let

$$\mathbf{P}_{g} = \frac{\mathbf{1}}{n_{f}} \sum_{i}^{T} \mathbf{P}_{j}$$
(4.4)

be the average of the conditional expected values defined in equation 4.3 for the particular set of n_t units included in trial t.

Then,

$$\sigma_{p_{t}}^{2} = E(p_{t} - P_{s} + P_{s} - P)^{2}$$

$$= E(p_{t} - P_{s})^{2} + E(P_{s} - P)^{2} + 2E(p_{t} - P_{s})(P_{s} - P)$$
(4.5)

The first term on the right-hand side of equation 4.5 is defined as the response variance, the second term is the sampling variance and the third term twice the covariance between response and sampling deviations.

In order to explore the response variance term further, let the response deviation be defined as:

$$d_{jt} = x_{jt} - P_j \qquad (4.6)$$

the difference between the observed value of the j-th unit on the t-th trial from the expected value of the responses for that unit over all trials. Then

$$\overline{\mathbf{d}}_{t} = \frac{1}{n_{t}} \sum_{j}^{n_{t}} \mathbf{d}_{jt}$$
(4.7)

is the mean of the response deviations over all units in a given trial. The response variance can then be written as:

$$E(p_{t} - P_{s})^{2} = E\left[\frac{1}{n_{t}} \sum_{j}^{n_{t}} (x_{jt} - P_{j})\right]^{2}$$
(4.8)

$$= \mathbf{E} \begin{bmatrix} \mathbf{n}_{t} \\ \mathbf{1}_{t} \\ \boldsymbol{\Sigma} \\ \mathbf{d}_{jt} \end{bmatrix}^{2}$$
(4.9)

= E
$$(\bar{d}_t)^2 = \sigma_{\bar{d}_t}^2$$
 (4.10)

So the response variance can be expressed as the variance of \overline{d}_t , the average of the response deviations.

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The response variance can be partitioned into two terms--a "simple response variance" term which shows the basic trial-to-trial variability in response averaged over all units and a term which reflects the correlation among response deviations of different units in the same trial.

Working with $\sigma_{d_t}^2$ as shown in equation 4.10 and expanding, we have

$$\sigma_{\vec{d}_{t}}^{2} = \mathbf{E} \begin{bmatrix} n_{t} & n_{t} \\ \frac{1}{n_{t}^{2}} \sum_{j} d_{jt}^{2} + \frac{1}{n_{t}^{2}} \sum_{j \neq k} d_{jt} d_{kt} \\ j \neq k \end{bmatrix}$$
(4.11)

For fixed sample size $n_t = n_t$,

$$\sigma_{d_{t}}^{2} = \frac{1}{n} E d_{jt}^{2} + \frac{n-1}{n} E d_{jt} d_{kt} \quad j \neq k$$
(4.12)

Let σ_d^2 be the variance of d $_{jt}$:

$$\sigma_{d}^{2} = E \left[d_{jt} - E d_{jt} \right]^{2}$$
(4.13)

Since $E d_{it} = 0$,

$$\sigma_{d}^{2} = \mathbf{E} d_{jt}^{2}$$
(4.14)

Also,
$$\rho_{d} = \frac{E d_{jt} d_{kt}}{\sigma_{d}^{2}}$$
 $j \neq k$ (4.15)

is the intraclass correlation among the response deviations in a given trial. Substituting equations 4.14 and 4.15 into 4.12, we have

$$\sigma_{\overline{d}_{t}}^{2} = \frac{\sigma_{d}^{2}}{n} + \frac{n-1}{n} \quad \sigma_{d}^{2} \quad \rho_{d} \qquad (4.16)$$

The first term on the right is the simple response variance, and the second term on the right reflects the correlation among the response deviations within a trial. This term is called the "within-trial covariance of response deviations." For most characteristics, it is the contribution from this term which is likely to be important. It is this term which we have tried to estimate in the RVS. It is this term that reflects the effect of interviewers and crew leaders. The interpretation an interviewer puts on a specific question, his possible misunderstanding of the training on certain questions, his tendency to accept nonresponses-all of these things and other peculiarities of interviewers and crew leaders will be reflected in this term for they affect the value of ρ_d .

In the results which follow, only the "within-trial covariance" term was computed, though we call it the response variance. The response variances produced are underestimates by the amount of the first term on the right-hand side of equation 4.16. However, as pointed out by Fellegi [4], the contributions to the response variances from the correlated component are several times as large as the simple response variance for all except basic population counts, such as the number of males, females, etc. In order to estimate the correlated component of the response variance, the method of interpenetrating samples as originally described by Mahalanobis [13] was used. As described in chapter 2, there were two interviewers working in each cluster of four or five EA's in the RVS. Therefore, in each of these clusters, each interviewer was assigned a random sample of the same population. Of every four units, two were assigned to interviewer 1 and the remaining two were assigned to interviewer 2. There are six possible ways to assign four units in this way. These patterns were applied successively to the group of four units to ensure a random sample for each interviewer. Figure 3 illustrates the concept of an interpenetrated experiment.

The figure shows that in each of the M clusters within a given area each interviewer of the pair was assigned half of the housing units. Two estimates for each cluster could then be formed, one for each interviewer.

Within each cluster of four or five EA's we wanted an estimate of the response variance. Within each cluster there were two interviewers. Of every four housing units two were assigned to one interviewer and the remaining two to the other interviewer. Then let subscript h refer to the h-th interviewer, subscript g refer to the g-th group-of-four housing units and the subscript j refer to the j-th housing unit within the g-th group-of-four. Also, k is 2, the number of interviewers assigned to the cluster; b is the total number of groups-of-four housing

Figure 3. An Interpenetration Experiment

Cluster	Housing units	Interviewers		
		1	2	
l	1 2 3	✓ ✓	* *	
2	1 2 3	*	4	
	•			
М	1 2 3 4	*	v v	

units in the cluster; and \overline{n} is 2, the number of units assigned to each interviewer within each group-of-four units. Using this notation,

$$\overline{\mathbf{x}}_{\dots} = \frac{\begin{pmatrix} \mathbf{k} & \mathbf{b} & \overline{\mathbf{n}} \\ \Sigma & \Sigma & \Sigma & \mathbf{x}_{hgj} \\ \mathbf{h} & \mathbf{g} & \mathbf{j} \\ \mathbf{k} & \mathbf{b} & \overline{\mathbf{n}} \\ \end{bmatrix}}{\mathbf{k} \mathbf{b} & \overline{\mathbf{n}}} = \frac{\begin{pmatrix} 2 & \mathbf{b} & 2 \\ \Sigma & \Sigma & \Sigma & \mathbf{x}_{hgj} \\ \mathbf{h} & \mathbf{g} & \mathbf{j} \\ \mathbf{h} & \mathbf{g} & \mathbf{j} \\ 4 & \mathbf{b} \\ \end{bmatrix}}$$
(4.17)

is the mean over all observations in the cluster, and

b
$$\overline{n}$$
 b 2
 $\Sigma \Sigma x_{hgj} \Sigma \Sigma x_{hgj}$ (4.18)
 $\overline{x}_{h..} = \frac{g j}{b \overline{n}} = \frac{g j}{2 b}$

is the mean for the h-th interviewer in the cluster.

Then, an estimator of the "total variance" of the mean for a given characteristic in the i-th cluster of EA's is:

$$C_{i} = \frac{1}{k-1} \frac{k}{\Sigma} (\bar{x}_{h..} - \bar{x}_{...})^{2}$$
 (4.19)

$$= \frac{2}{\Sigma} (\overline{\mathbf{x}}_{h..} - \overline{\mathbf{x}}_{...})^2 \qquad (4.20)$$

The "total variance" is different from the usual variance in that it not only includes the sampling variance and the simple response variance, but also the correlated component of the response variance.

Expanding the expression in equation 4.20,

$$C_{i} = \frac{1}{8b^{2}} \begin{bmatrix} b & 2 & b & 2 \\ (\Sigma & \Sigma & x_{1gj})^{2} & + & (\Sigma & \Sigma & x_{2gj'})^{2} \\ g & j & g & j' \\ - & 2 & \Sigma & \Sigma & x_{1gj} & \Sigma & \Sigma & x_{2gj'} \\ g & j & g & j' & g & j' \end{bmatrix}$$
(4.21)

In order to express the results in terms of response deviations and sampling deviations, define

 $d_{hgj} = x_{hgj} - P_{gj}$

 \mathbf{a} nd

$$\Delta_{gj} = P_{gj} - X$$

where \overline{X} is the population mean. Thus, d_{hgj} is the response deviation as defined following equation 4.6 and Δ_{gj} is the sampling deviation, the difference between the expected value of the response for a given unit over all trials and \overline{X} . Each observation can be expressed as the sum of a response deviation, a sampling deviation, and a population mean as follows:

 $\mathbf{x}_{hgj} = \mathbf{d}_{hgj} + \Delta_{gj} + \overline{\mathbf{X}}$ (4.23)

(4.22)

In order to simplify the expression we would get by substituting in equation 4.21 the value of x_{hgj} as given in equation 4.23, let us look at some expected values.

 $E d_{hgj}^2 = \sigma_d^2$, the simple response variance as defined in equation 4.14.

 $E d_{hgj}d_{hgk} = E d_{hgj}d_{hmj} = E d_{hgj}d_{hmk} = \rho_d \sigma_d^2$, the correlated component, as shown in equation 4.15.

$$\Delta^2_{gj} = \sigma^2_s$$
, the sampling variance.

 $E \Delta_{gj} \Delta_{gk} = E \Delta_{gj} \Delta_{mj} = E \Delta_{gj} \Delta_{mk} = \delta \sigma_s^2$, the correlation among the sampling deviations of the elements in the same sample multiplied by the sampling variance.

Using these expected values, we have:

$$8b^{2} E(C_{1}) = 4b\sigma_{d}^{2} + 4b\rho_{d}\sigma_{d}^{2} + 8b(b-1)\rho_{d}\sigma_{d}^{2} + 4b\sigma_{s}^{2} \quad (4.24)$$
$$- 4b\delta\sigma_{s}^{2} + 2E \begin{bmatrix} b & 2 \\ \Sigma & \Sigma & d_{1gj} & \Sigma \\ g & j & g \end{bmatrix}$$
$$+ 2E \begin{bmatrix} b & 2 \\ \Sigma & \Sigma & d_{2gj'} & \Sigma & \Sigma \\ g & j' & g & j' \end{bmatrix} - 2E \begin{bmatrix} b & 2 \\ \Sigma & \Sigma & d_{1gj} & \Sigma & \Delta_{2gj'} \\ g & j & g & j' \end{bmatrix}$$

$$-2E \begin{bmatrix} b & 2 & b & 2 \\ \Sigma & \Sigma & d_{1gj} & \Sigma & \Sigma & A_{gj'} \\ g & j & g & j' \end{bmatrix} - 2E \begin{bmatrix} b & 2 & b & 2 \\ \Sigma & \Sigma & d_{2gj'} & \Sigma & \Sigma & A_{gj} \\ g & j & g & j \end{bmatrix}$$

To simplify the expression in equation 4.24 we need to make some assumptions. First,

$$\mathbf{E}\begin{bmatrix}\mathbf{b} & 2 & \mathbf{b} & 2\\ \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \mathbf{d}_{1gj} & \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \mathbf{d}_{2gj'}\\ \mathbf{g} & \mathbf{j} & \mathbf{g} & \mathbf{j}' \end{bmatrix} = \mathbf{0}$$
(4.25)

if we assume that the covariance between the response deviations obtained by different interviewers is zero. This term may be different from zero if both interviewers are subject to the influence of the same crew leader. If this term is not equal to zero and there is a positive correlation among the response deviations of interviewers trained and supervised by the crew leader, then the effect of this term can be seen by comparing the values of the response variance for the clusters containing the crew leader effect with those clusters not containing the crew leader effect. In those clusters where each interviewer was supervised by a different crew leader, we will not be subtracting a positive term from the estimate of C_i if we assume that this term is zero. In chapter 7, these comparisons are made and for some items, this term is not zero. However, for the purpose of expressing the expected value of the total variance in simple terms, assume this covariance is zero.

The next assumption is:

$$\mathbf{E}\begin{bmatrix}\mathbf{b} & 2 & \mathbf{b} & 2\\ \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \mathbf{d}_{1gj} & \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \boldsymbol{\Delta}_{gj'}\\ \mathbf{g} & \mathbf{j} & \mathbf{g} & \mathbf{j'} \end{bmatrix} = \mathbf{E}\begin{bmatrix}\mathbf{b} & 2 & \mathbf{b} & 2\\ \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \mathbf{d}_{2gj'}, & \boldsymbol{\Sigma} & \boldsymbol{\Sigma} & \boldsymbol{\Delta}_{gj}\\ \mathbf{g} & \mathbf{j'} & \mathbf{g} & \mathbf{j} & \mathbf{g} \end{bmatrix} = \mathbf{0}$$
(4.26)

Equation 4.26 expresses the assumption that the response deviations obtained by one interviewer are uncorrelated with the sampling deviations in the sample of the other interviewer. This assumption seems to be fairly realistic.

The third assumption is:

$$E\begin{bmatrix}b&2&b&2\\\Sigma&\Sigma d_{Igj}&\Sigma&\Sigma \Delta_{gj}\\g&j&g&j\end{bmatrix} = E\begin{bmatrix}b&2&b&2\\\Sigma&\Sigma d_{2gj'}&\Sigma&\Sigma \Delta_{gj'}\\8&j'&8&j'\end{bmatrix} = 0 \quad (4.27)$$

Here we have assumed that the response and sampling deviations obtained by one interviewer are uncorrelated. Hansen, Hurwitz, and Bershad [6] point out that this term may not be zero since the responses an interviewer obtains may be influenced by the other units in the sample. Fellegi [4] gave some estimates of the effect of this correlation. However, we have assumed that the effect of this term is likely to be small in comparison with the contribution of the leading terms in equation 4.24. So, for the purpose of simplification, we assume that this covariance is zero.

Using these assumptions, the expected value of C_i is:

$$E(C_{i}) = \frac{\sigma_{d}^{2}}{2b} \left[1 + \rho_{d}(2b-1)\right] + \frac{\sigma_{s}^{2}}{2b} \left[1 - \delta\right]$$
(4.28)

where ρ_d is the correlation among response deviations and δ is the correlation among the sampling deviations of the elements in the same sample.

Since 2b is the number of units in the cluster for each of the two interviewers, let 2b equal n. Then,

$$E(C_{i}) = \frac{\sigma_{d}^{2}}{n} \left[1 + \rho_{d}(n-1)\right] + \frac{\sigma_{s}^{2}}{n} \left[1 - \delta\right]$$
(4.29)

We should like to have an estimator for the sampling variance so that the difference between it and C_1 could be used as an estimator of the correlated component of the response variance. Given

$$\overline{\mathbf{x}}_{\mathrm{hg.}} = \frac{\overline{\mathbf{n}}}{\sum_{j}} \frac{\mathbf{x}_{\mathrm{hgj}}}{\overline{\mathbf{n}}}$$
(4.30)

is the mean of the h-th interviewer in the g-th group-offour units, then an estimator of the sampling variance in the i-th cluster is:

$$D_{i} = \frac{\begin{array}{c}k & b & \overline{n} \\ \Sigma & \Sigma & \Sigma \\ h & g & j \end{array}}{k & b^{2} & \overline{n}(\overline{n}-1)\end{array}}$$
(4.31)

Using the same values of k, b and \overline{n} as given on page 12 we have

$$D_{i} = \frac{\begin{pmatrix} 2 & b & 2 \\ \Sigma & \Sigma & \Sigma \\ h & g & j \\ \end{pmatrix}}{\frac{h & g & j}{4b^{2}}}$$
(4.32)

$$= \frac{\frac{2}{\sum} \frac{b}{\sum} (x_{hg1} - x_{hg2})^{2}}{\frac{h}{8b}^{2}}$$
 (4.33)

Substituting the expression for x_{hgj} as given in equation 4.23 in terms of response and sampling deviations and using the expected values given on page 13, we have

$$8b^{2} E(D_{i}) = 4b\sigma_{d}^{2} + 4b\sigma_{s}^{2} - 4b\rho_{d}\sigma_{d}^{2} - 4b\delta\sigma_{s}^{2}$$
$$+ 2E\left[\sum_{h=g}^{2}\sum_{g=1}^{b}d_{hgl}\Delta_{gl} - \sum_{h=g}^{2}\sum_{g=1}^{b}d_{hgl}\Delta_{g2}\right]$$
$$- 2E\left[\sum_{h=g}^{2}\sum_{g=1}^{b}d_{hg2}\Delta_{g1} - \sum_{h=g}^{2}\sum_{g=1}^{b}d_{hg2}\Delta_{g2}\right] \qquad (4.34)$$

But $E d_{hgl} \Delta_{gl} = E d_{hgl} \Delta_{g2}$ and $E d_{hg2} \Delta_{g1} = E d_{hg2} \Delta_{g2}$ so that the last two terms in equation 4.34 are zero and

$$E(D_{i}) = \frac{\sigma^{2}}{n} [1 - \rho_{d}] + \frac{\sigma^{2}}{n} [1 - \delta]$$
 (4.35)

The expected value of the estimate of the correlated component of the response variance is given by

$$E(C_{i} - D_{i}) = \frac{\sigma_{d}^{2}}{n} [1 + \rho_{d}(n - 1]] + \frac{\sigma_{s}^{2}}{n} [1 - \delta] - \frac{\sigma_{d}^{2}}{n} [1 - \rho_{d}] - \frac{\sigma_{s}^{2}}{n} [1 - \delta] = \rho_{d} \sigma_{d}^{2}$$
(4.36)

Thus $C_i - D_i$ is an unbiased estimator of the correlated component of the response variance under the assumptions we have made.

The estimates of the response variance apply to the particular set of conditions, G, that underlie this survey. In particular, these conditions include the circumstance that the unit of observation in the experiment as well as in the census was the household, and not the person. Therefore the number of persons reported is also subject to response variability. As a result, the response variance estimates of complementary characteristics like native and foreign-born are not equal, which they would be if the units of observation were persons, or as they would be if we were considering a pair of complementary <u>household</u> characteristics (i.e., nativity of household head).¹

We have calculated the estimates C_i , D_i , and $C_i - D_i$ for each cluster (i) of EA's. The individual estimates are unreliable since each estimate is based on only one degree of freedom. Therefore, we have averaged the values over all clusters in order to increase the reliability.

For each cluster, there is an unbiased estimate $\overline{\mathbf{x}}_{i}$ of

the mean per household in the i-th cluster. Then an unbiased estimate of the average value over all clusters is

$$\overline{\mathbf{x}} = \frac{\Sigma \mathbf{N}_{1} \cdot \overline{\mathbf{x}}_{1}}{\Sigma \mathbf{N}_{1}}$$
(4.37)

¹In the appendix, an illustration in terms of a simple probability model is given, for which even the simple response variance is different for complementary variables. and its variance is

$$\operatorname{Var}(\overline{\mathbf{x}}) = \frac{\Sigma N_i^2 \operatorname{Var}(\overline{\mathbf{x}}_i)}{\left(\Sigma N_i\right)^2}$$
(4.38)

The same relationship holds for each of the components of the variance. Now we want estimates of the response **variance** for an average-sized cluster of size \overline{N} where

$$\overline{N} = \frac{\frac{L}{\Sigma} N_i}{L}$$

Suppose all the clusters were the same size. Then

$$\operatorname{Var}(\overline{\mathbf{x}}) = \frac{1}{L^2 \overline{N}^2} \left[L \ \overline{N}^2 \ \operatorname{Var}(\overline{\mathbf{x}}) \right]$$
(4.39)

so that

$$\operatorname{Var}(\overline{\mathbf{x}}) = \operatorname{L} \operatorname{Var}(\overline{\mathbf{x}})$$
 (4.40)

If we now replace $Var(\overline{x})$ by the estimate of the correlated **Component** of the response variance given by C_i - D_i , we have

$$\operatorname{Var}(\overline{\mathbf{x}}_{\overline{\mathbf{N}}}) = \mathbf{L} \frac{\Sigma \ \mathbf{N}_{i}^{2} \ (\mathbf{C}_{i} - \mathbf{D}_{i})}{(\Sigma \ \mathbf{N}_{i})^{2}}$$
(4.41)

as a rough estimate of this component for a cluster of size \overline{N} . This is the estimate of the response variance which was used in the RVS. Since it was convenient to work with relative errors in the analysis, we estimated the relvariance as follows:

relvariance
$$\approx \frac{Var(\bar{x})}{\bar{p}_{t}^{2}}$$
 (4.42)

where \overline{p}_t is the estimated proportion of the population with the specific characteristic for an average-sized cluster.

Because it was possible for the estimates of the response variance to be negative, a "restricted estimator" was used. Whenever the relvariance estimated by equation 4.42 was negative it was replaced by zero. As stated by Kendall and Stuart [10], "For practical purposes, a negative point estimate of a non-negative quantity is inadmissible, and is usually replaced by the estimate zero (although this removes the unbiassedness of the estimator)." It was shown by Benjamin J. Tepping [24] that the mean-square error of the restricted estimator is smaller than that of the unbiased estimate. Therefore, in chapter 7, the values of the relvariance that are shown are those produced by using equation 4.42 or zero.

Chapter 5--DISTRIBUTION OF THE ESTIMATORS FOR THE TOTAL VARIANCE AND SAMPLING VARIANCE

When the computation of C_i , the estimate of the total variance, D_i , the estimate of the sampling variance, and C_i-D_i , the estimate of the response variance, was completed there were available 384 estimates of these kinds for the clusters without crew leader effect and 390 estimates for the clusters with crew leader effect. It was found that well over half of the values of C_i-D_i were negative for most of the characteristics being studied. To find out the reasons for this, an investigation into the distribution of the estimators was begun.

Express C_1 as follows:

$$C_{1} = \frac{1}{2} \left[\overline{x}_{1..} - \overline{x}_{2..} \right]^{2}$$
 (5.1)

where $\overline{\mathbf{x}}_{1}$ is the mean for interviewer 1 in the i-th cluster of EA's and $\overline{x}_{2...}$ is the mean for interviewer 2 in that same cluster. Then $\overline{x}_{1,..}$ can be thought of as a single observation from the distribution of the sample mean for a specified characteristic. Similarly $\overline{x}_{2...}$ is a single observation from the distribution of the sample mean. It has been shown, for infinite populations, that the distribution of the sample mean tends to a normal distribution for any population with a finite variance. However, in this case, we sampled without replacement from a finite population. Cochran [1] discusses the validity of the normal approximation and gives references to work relating to sampling without replacement from finite populations. (See Hajek [5], Erdos and Renyi [3] and Madow [12] for discussions of the conditions under which the distribution of the sample mean is asymptotically normal.)

We shall assume that the distribution of the sample mean tends to normality. Let us view $\overline{x}_{1..}$ and $\overline{x}_{2..}$ as selections from a normal distribution with mean \overline{X} and variance $\sigma_{\overline{x}_{hgj}}^2$ /n, assuming that we have a simple random sample. Also, for purposes of simplification, let us assume that $\overline{x}_{1..}$ and $\overline{x}_{2..}$ are independent observations from that distribution. (Except for the possible influence of the crew leader in clusters where both interviewers worked for the same crew leader, this assumption is approximately met.) Then,

$$\operatorname{var}(\overline{x}_{1..} - \overline{x}_{2..}) = \operatorname{var}(\overline{x}_{1..}) + \operatorname{var}(\overline{x}_{2..}) \quad (5.2)$$
$$= \frac{2\sigma_{x}^{2}}{\frac{\log n}{n}}$$

and

$$\operatorname{var} \frac{1}{\sqrt{2}} \left(\overline{\mathbf{x}}_{1..} - \overline{\mathbf{x}}_{2..} \right) = \frac{\sigma_{\mathbf{x}}^2}{n} = \sigma_{\overline{\mathbf{x}}}^2$$
(5.3)

Let W be defined as follows:

$$W = \frac{(\bar{x}_{1..} - \bar{x}_{2..})}{\sqrt{2} \sigma_{\bar{x}_{hgj}}}$$
(5.4)

Then W is normally distributed since the difference between two normally distributed variables is also normally distributed and a normally distributed variable multiplied by a constant is still normally distributed. W has a zero mean and unit variance. Therefore,

$$W^{2} = \frac{(\bar{x}_{1..} - \bar{x}_{2..})^{2}}{2 \sigma_{\bar{x}_{hgj}}^{2}}$$
(5.5)

is distributed as a Chi-square with one degree of freedom (X_1^2) .

It follows from this that the statistic defined as C_i in equation 5.1 is distributed approximately as $\sigma_{\overline{x}\,hgj}^2 \chi_1^2$. By dividing each C_i by $\sigma_{\overline{x}\,hgj}^2$, the resulting distribution is $\chi_{1^*}^2$.

The distribution of the sampling variance D_i , as given in equation 4.33, is slightly more difficult to obtain. Let

$$U_{1} = X_{111} - X_{112}$$
(5.6)

where the \mathbf{x}_{hgj} 's are assumed to be independent observa-

tions from the same two-point distribution. Of course, this assumption is not met when there is a constant interviewer effect. The results described in chapters 6 and 7 show that there is an interviewer effect. However, in order to compare the observed results with the results expected in the absence of interviewer effect, this assumption of independent observations is made. If the x_{hgj} 's are independent, then each x_{hgj} takes on the value 1 with probability p and the value 0 with probability q where q = 1-p.

 U_1^2 is also a 0-1 variate with probability 2 pq of having the value 1 and probability $p^2 + q^2$ of having the value 0.

Now, let $U_2,\ U_3,\ \ldots \ U_n$ be defined in the same way as U_1 so that each U_1 is independent of everyother U. Then,

$$\mathbf{B} = \mathbf{U}_{1}^{2} + \mathbf{U}_{2}^{2} + \ldots + \mathbf{U}_{n}^{2}$$
(5.7)

has a binominal distribution with parameters 2 pq and n. B can be written as follows:

$$B = \sum_{h=g}^{2} \sum_{g=1}^{b} (x_{hg1} - x_{hg2})^{2}$$
(5.8)

Then D_i , as it is defined in equation 4.33, can be expressed as:

$$D_{i} = \frac{B}{8b^{2}} = \frac{B}{2n^{2}}$$
(5.9)

Now both C_i and D_i are expressed in terms of distributions with known means and variances. C_i is distributed as $\sigma_{\overline{X}_{hgj}}^2 = X_1^2$. Since x is an observation from a two-

point distribution,
$$\sigma \frac{2}{\mathbf{x}}$$
 is pq/n. So
 $C_i = \frac{pq}{n} \times \frac{2}{1}$
(5.10)

Now,
$$E(C_i - D_i) = E(C_i) - E(D_i)$$

$$= \frac{pq}{n} E(X_1^2) - \frac{1}{2n^2} E(B)$$

$$= \frac{pq}{n} (1) - \frac{1}{2n^2} (2 npq)$$

$$= 0$$
(5.11)

This emphasizes the fact that this model does not describe the real world, for the conditions of the model imply that $\rho_d \sigma_d^2 = 0$. If, further, C_i and D_i are uncorrelated,

$$\operatorname{Var} (\mathbf{C}_{i} - \mathbf{D}_{i}) = \operatorname{Var} (\mathbf{C}_{i}) + \operatorname{Var} (\mathbf{D}_{i})$$
$$= \frac{p^{2} q^{2}}{n^{2}} \operatorname{Var} (X_{1}^{2}) + \frac{1}{4n^{4}} \operatorname{Var} (\mathbf{B})$$
$$= \frac{p^{2} q^{2}}{n^{2}} (2) + \frac{1}{4n^{4}} n (2pq) (1-2pq)$$
$$= \frac{p^{2} q^{2} (2n-1)}{n^{3}} + \frac{pq}{2n^{3}}^{1}$$
(5.12)

The reason for the occurrence of so many negative estimates of the response variance is shown by consideration of these distributions. $C_i / \sigma_{\overline{x} hgj}^2$ is distributed as X_1^2 . $D_i / \sigma_{\overline{x} hgj}^2$ is distributed as B/2npq.

Then,

$$E(C_{i} - D_{i}) / \sigma^{2} = E[X_{1}^{2} - B/2npq] = 0$$

$$\operatorname{Var}(C_{i} - D_{i}) / \sigma_{\bar{x}_{hgj}}^{2} = \operatorname{Var} X_{1}^{2} + \operatorname{Var}(B/2npq)$$

= 2 + $(\frac{1}{2pq} - 1)\frac{1}{n}$

The average number of units assigned to each interviewer in a cluster was 165. Therefore, the variance of

$$D_i / \sigma_{\overline{X}_{hgj}}^2$$
 is $(\frac{1}{2pq} - 1)\frac{1}{n}$ which is about 2/165 or .012 for

an item where p is about .20. For a p of .05, the variance of $D_i/\sigma_{\overline{x}\ hgj}^2$ is about .05 and its smallest value for a p of .50 is about .006. Since this variance is so small in comparison with the variance of $C_i/\sigma_{\overline{x}\ hgj}^2$ which is 2, consider the distribution of $D_i/\sigma_{\overline{x}\ hgj}^2$ as concentrated at the point 1. Then, in the $C_i/\sigma_{\overline{x}\ hgj}^2$ distribution, the probability is about .683 that an observation is less than 1. (See cumulative tables of X_1^2 distribution.) Therefore, about .683 or roughly two-thirds of the estimates of $C_i - D_i$, the correlated component of the response variance, would be negative for an item which had no interviewer effect, i.e. for which $\rho_d \sigma_d^2 = 0$.

A study [14] was conducted in which three items were selected for which the distributions of $C_i / \sigma \frac{2}{X_{hgj}}$ and $D_i / \sigma \frac{2}{X_{hgj}}$ were compared with the distributions expected

in the absence of any interviewer effect. The three items were: a basic population count, expected to have no interviewer effect; a mobility item, expected to show some interviewer effect; and a nonresponse item, expected to show a high degree of interviewer effect. Some of the conclusions drawn from this study are:

1. There were certain basic population counts which were expected to show no interviewer effect. For these items, the distribution of $C_i / \sigma_{\overline{X}_{hgj}}^2$ approximately followed an X_1^2 distribution and the distribution of $D_i / \sigma_{\overline{X}_{hgj}}^2$

was very highly concentrated about 1. About two-thirds of the estimates of the response variance for an item of this type were negative.

2. For items which were not basic population counts, the average of the estimates of the response variance was usually positive. About the same number of pairs of interviewers contributed to this positive effect from both the clusters with and the clusters without the crew leader effect.

3. For items which were not basic population counts, the distribution of $C_i/\sigma_{\overline{x}\ hgj}^2$ followed a X_1^2 distribution approximately only after some "extreme" values were removed. More estimates were "extreme" for a non-response item that for other types of items. This result was verified by other studies of interviewer effect which showed that nonresponse items were heavily influenced by interviewers.

 $^{^1\,\}rm It$ is evident from equation 5.12 that the variance of (C $_i$ - $D_i)$ depends on the sample size within cluster. Since the sample size varies from 42 to 426 households, the value of the variance will vary from cluster to cluster, even for the same characteristic.