APPENDIX

It is a natural reaction for the reader to assume that the response variances for complementary variables (like "native" and "foreign-born") should be equal. That this is not necessarily true is shown by the following example.

Suppose that an interviewer is assigned a unit which contains \( n \) persons of whom \( X_1 \) have a given characteristic (e.g., native) and the remainder \( X_2 \) have the complementary characteristic (e.g., foreign-born). Let \( p \) denote the probability that the interviewer will record any given person in the unit. Let \( \pi_1 \) denote the conditional probability that the interviewer will classify a recorded person as having the first characteristic, when in fact that person does have the first characteristic, and let \( \pi_2 \) denote the conditional probability that the interviewer will classify a recorded person as having the first characteristic, when in fact that person has the second characteristic. (I.e., \( \pi_1 \) is the probability that a native person is classified as native, and \( \pi_2 \) is the probability that a foreign-born person is classified as native.)

Let \( \xi_1 \) denote the number of persons classified by the interviewer in the two classes. Thus

\[
\begin{align*}
\mathbb{E} \xi_1 &= p \pi_1 X_1 + p \pi_2 X_2 \\
\mathbb{E} \xi_2 &= p (1- \pi_1) X_1 + p (1- \pi_2) X_2
\end{align*}
\]

and

\[
\begin{align*}
\text{Var} \xi_1 &= p \pi_1 (1-p \pi_1) X_1 + p \pi_2 (1-p \pi_2) X_2 \\
\text{Var} \xi_2 &= p (1- \pi_1)^2 (1-p(1- \pi_1)) X_1 + p (1- \pi_2)^2 (1-p(1- \pi_2)) X_2
\end{align*}
\]

Since, clearly, no sampling of the population is involved here, these variances are simple response variances.

We may now note that, if \( p = 1 \) (i.e., if there is no response variability in recording the number of persons), then \( \text{Var} \xi_1 = \text{Var} \xi_2 = \pi_1 (1- \pi_1) X_1 + \pi_2 (1- \pi_2) X_2 \). If, however, \( p < 1 \) then it can be shown that \( \text{Var} \xi_1 > \text{Var} \xi_2 \) if and only if

\[
(2 \pi_1-1)X_1 + (2 \pi_2-1)X_2 > 0.
\]

In the case of nativity, \( X_1 \) is a much larger number than \( X_2 \), and \( \pi_1 \) is very nearly 1. Thus the inequality condition is satisfied quite strongly for these characteristics. To illustrate, take

\[
\begin{align*}
p &= .95 \\
\pi_1 &= .995 \\
\pi_2 &= .20 \\
X_1 &= .95N \\
X_2 &= .05N
\end{align*}
\]

where \( N \) is the total number of persons in the unit. Thus

\[
\frac{\text{Var} \xi_1}{\text{Var} \xi_2} = 4.2.
\]